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FOUNDATION VIBRATIONS

By F. E. Richart, Jr.,[1] F. ASCE

SYNOPSIS

Theoretical solutions for the vibrations of foundations resting on soil have been studied and evaluated. For vertical vibrations of a machine foundation, it is usually necessary to use the theory for an oscillator resting on an elastic semi-infinite body because of the important effects introduced by oscillations of the soil. This effect becomes of less importance if the force of the oscillator varies with the frequency of rotation. The use of Sung's theory for vertical foundation vibrations is outlined and curves are presented which may be used for analysis or design.

By comparison of the ordinary theory of vibration, which considers the spring to be weightless, to the theory for an oscillator on a semi-infinite elastic body, it is shown that the ordinary theory is usually satisfactory for determining the natural frequency for the rocking and sliding modes of vibration of the foundation.

In order to evaluate the dynamic characteristics of a foundation, it was found necessary to determine the shear modulus and Poisson's ratio of the underlying soil. Typical values of these soil constants are given and methods for establishing them at a particular site are described. The shear modulus of a granular soil should increase approximately with the 1/3 power of the confining pressure.

Two examples are included to show the details of application of the theoretical methods. A diagram is also included which illustrates the possibility of

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stiffening the foundation, by the use of point-bearing piles to rock, in order to increase the natural frequency of vertical vibration.

INTRODUCTION

Oscillating or rotating machine components are a source of periodic impulses. Because large machines are usually supported directly on the soil in a manner that permits a direct transmission of these periodic impulses into the soil, the design of the machine foundation involves the problem of soil dynamics. When the natural frequency of a soil-foundation system happens to coincide with the frequency of the exciting forces generated by the machine, excessive vibration amplitudes may occur which can lead to structural damage or operational failure of the machine. Periodic or intermittent wind forces on tall structures may also set up vibrations of the foundations for a sufficient time to cause damage.

The type of foundation considered in this study consists of a massive block resting directly on the soil. The block is freely supported and has six degrees of freedom in vibration: It may vibrate in vertical, longitudinal, or lateral translation, or it may oscillate in the rotational modes of rocking, pitching, or yawing. These motions are illustrated in Fig. 1. In general, the vertical translation and the rocking types of vibration occur most often in machine foundations.

The first approximation to the study of foundation vibrations considered the vibrating system to behave as a single mass supported by a weightless spring and subjected to a viscous damping. As a result of an extensive series of tests made by DEGEBO (Deutsche Ferschungagesellschaft für Bodenmechanik) it was found that the simple damped mass-spring theory was not adequate to explain the test results obtained from vertical motions of an oscillator resting on soil. A. Gertwig, G. Früh, H. Lorenz, and co-workers found (1)[2] that it was necessary to include consideration of an "in-phase" mass of soil which oscillated with the foundation to made the simple theory agree with test results. Unfortunately, test data prove that the computed in-phase soil mass is not a constant quantity, even for a particular foundation, because it varies with both the static and dynamic loading. In the intervening years since these test results became available, repeated attempts have been made to provide rules for determining the weight of this in-phase mass, with negligible success.

In 1936, E. Reissner presented (2) an analytical solution to the problem of vertical motion of an oscillator resting on an elastic semi-infinite body. His solution included the dynamic behavior of the elastic body, and represented the oscillator by a pulsating pressure uniformly distributed over a circular area of contact. More recently, both T. Y. Sung (3) and P. M. Quinlan (4) have extended Reissner's treatment to cover different contact pressure distributions between the oscillator and elastic body, and R. N. Arnold, G. N. Bycroft, and G. B. Warburton (5), and I. Toriumi (6) have considered different modes of vibration. The theories based on the treatment of an elastic semi-infinite body have shown relatively good agreement with test results.

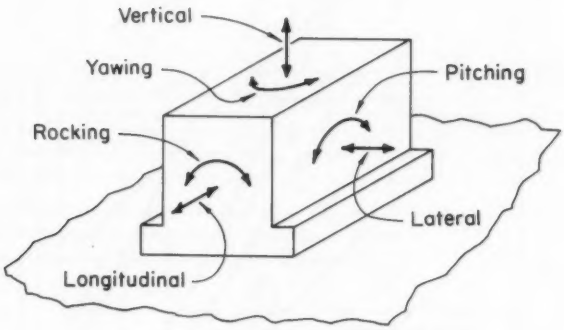
It is the purpose of this paper to examine briefly some of the theoretical procedures which are available for analysis and design of foundations subjected to vibratory loads. Sand compaction by vibrations and the resulting settlements of foundations is a separate problem which will not be considered here.

2 Numerals in parentheses, thus (1), refer to corresponding items in the Bibliography.

Notation.—The letters and symbols used in the test are defined as they appear and are assembled in Appendix I.

LIMITATIONS OF VIBRATION AMPLITUDE

It is obviously impossible to eliminate the oscillating motion completely from a foundation which is subjected to significant periodic impulses. The designer can only attempt to reduce the foundation vibration to a magnitude which is tolerable at the operating frequency for the design conditions. In general, the permissible amplitude of vibration is decreased as the frequency is increased. Thus, no value of allowable amplitude should be considered as a design criterion unless the frequency is also specified. In cases where the design specifications do not include permissible amplitudes of vibration, the



Translational Modes	Rotational Modes
Vertical	Rocking
Longitudinal	Pitching
Lateral	Yawing

FIG. 1.—SIX MODES OF VIBRATION FOR A FOUNDATION

information given in Fig. 2 may be used as a guide. It is important to note from Fig. 2 that the vibration amplitude which is designated as the limit for machines and machine foundations is approximately one hundred times that which is barely noticeable to persons.

DAMPED FORCED VIBRATIONS FOR ONE DEGREE OF FREEDOM

It is useful to review briefly the simple case of damped forced vibrations of a single mass supported by a weightless spring. As shown in Fig. 3 the mass has a weight, W_O, which is set into oscillation by a periodic force Q. The motion is restrained by the inertia of the mass, by the reaction of the spring, and by the viscous damping force. The periodic exciting force is defined by

$$Q = Q_1 \sin \left(2 \pi f_1 t\right) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

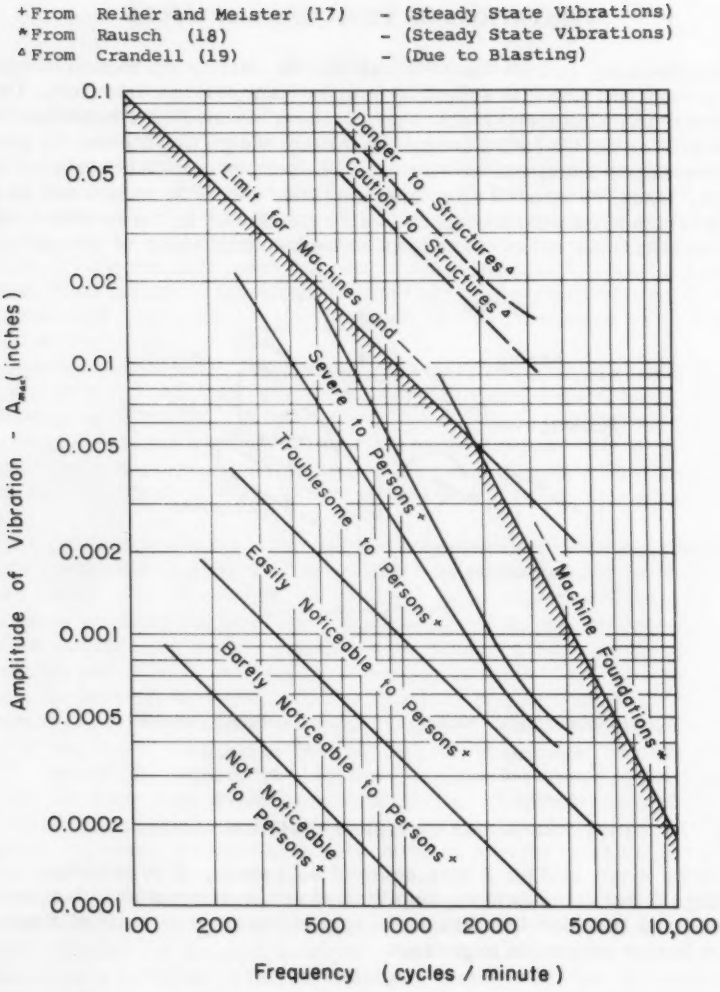
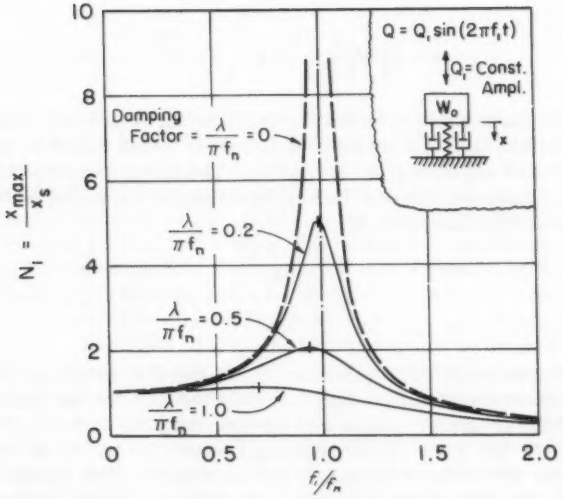
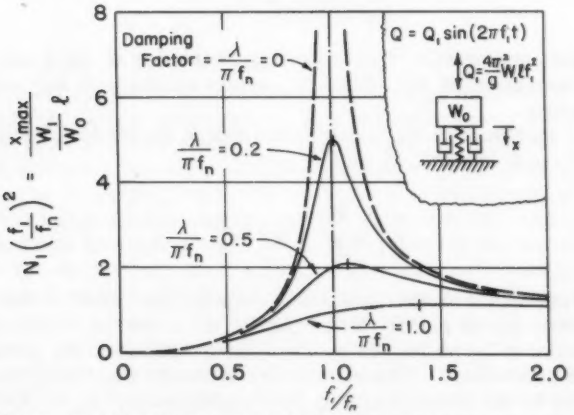


FIG. 2.—ALLOWABLE VERTICAL VIBRATION AMPLITUDE FOR A PARTICULAR FREQUENCY OF VIBRATION



(a) For Constant Amplitude Exciting Force



(b) For Exciting Force Dependent Upon Exciting Frequency

FIG. 3.—AMPLITUDE—FREQUENCY RELATIONS FOR DAMPED FORCED VIBRATION OF A MASS—SPRING SYSTEM

in which Q_1 may be either a constant or a function of the exciting frequency, f_1, and t denotes time. For the case where Q_1 is a constant and thus independent of the exciting frequency, the resulting displacement of the mass is given by

$$x = x_s N_1 \sin\left[(2\pi f_1 t) - \alpha\right] \quad \ldots\ldots\ldots\ldots\ldots (2)$$

in which α is the phase angle between the exciting force and the resulting motion of the mass, and x_s is the static displacement which would be produced by a force of magnitude Q_1 acting on the spring. The force required to compress the spring a unit distance defines the spring constant, c_s, which is then used to evaluate the static displacement by

$$x_s = \frac{Q_1}{c_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The displacement magnification factor, N_1, in Eq. 2 is shown as the ordinate in Fig. 3(a) and the curves on this diagram show the manner in which N_1 varies as a function of the frequency ratio and of the damping factor, $\lambda/(\pi f_n)$. The frequency ratio is the ratio of the exciting frequency, f_1, to the natural frequency, f_n, of free vibration of the mass-spring system. The natural frequency of free vibration without damping of the mass-spring system is given by

$$f_n = \frac{1}{2\pi} \sqrt{\frac{c_s g}{W_o}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which g is the acceleration of gravity. The derivation of Eq. 2 and a discussion of the preparation of Fig. 3(a) is readily available in any book on elementary vibrations.

For rotating machinery with unbalanced weights, the exciting force is a function of the exciting frequency, f_1, or

$$Q_1 = \frac{W_1}{g} l\, 4\pi^2 f_1^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which W_1 is the weight of the unbalanced rotating part and l is the eccentric radius from the center of gravity of the part to the center of rotation. By introducing the value of Q_1 as defined by Eq. 5, the solution for the displacement, x, can again be determined. The results are shown in Fig. 3(b), in which the ordinate is equal to the ordinate of Fig. 3(a) multiplied by $(f_1/f_n)^2$. The value of maximum vibration amplitude is

$$x_{max} = N_1 \left(\frac{f_1}{f_n}\right)^2 \frac{W_1}{W_o} l \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which W_o represents the static weight of the mass. It should be noted that on Fig. 3(b) the curves all approach an ordinate value of 1.0 as the frequency

ratio becomes large. Thus, for large values of the frequency ratio, the force transmitted through the spring to the rigid base is

$$(F)_{f_1/f_n \to \infty} = c_s x_{max} = c_s \frac{W_1}{W_o} 1 \quad \ldots\ldots\ldots\ldots (7)$$

Another important quantity which can be defined by consideration of Figs. 3(a) and 3(b) is the term "resonance," used to denote the condition of vibration corresponding to a large increase in amplitude. For undamped forced vibrations this condition occurs at $f_1/f_n = 1.0$. When the exciting force has a constant amplitude regardless of frequency, the curves representing damped forced vibrations in Fig. 3(a) show the maximum amplitude magnification at values of f_1/f_n less than 1.0. For small values of damping the amplitude peaks occur at frequency ratios so close fo $f_1/f_n = 1.0$ that the difference is usually ignored. However, when the damping factor, $\lambda/(\pi f_n)$, equals 1.0, the peak is at $f_1/f_n = 0.7$. The other condition, for which the exciting force is a function of the exciting frequency, establishes peak amplitudes at a value of f_1/f_n greater than 1.0, which for the damping factor of 1.0 results in a peak at $f_1/f_n = 1.4$. This shows the effect of damping in shifting the frequency for maximum amplitude of vibration away from the "natural" frequency of the foundation. It also demonstrates that if Fig. 3(a) is used to determine the frequency for peak amplitude when the exciting force is actually frequency dependent, the selected frequency may be as low as one half the correct value.

OSCILLATORS ACTING ON THE SURFACE OF A SEMI-INFINITE ELASTIC BODY

Elastic Waves.—The vibrations examined previously have considered a closed system in which the periodic motion was confined to the mass and spring, with the spring supported by an immovable base. An actual foundation which has been set into vibratory motion by a periodic force becomes the source of periodic impulses which proceed into the subgrade in radial directions, similar to a sound wave. In the course of transmitting waves the particles of the subgrade also undergo periodic motion, but only at particular locations do these motions correspond to the motion of the foundation. Directly below the foundation base the subgrade material moves with the foundation and is "in phase" with the foundation motion. At a greater distance a zone of the subgrade moves opposite to the foundation motion and is designated as "180° out of phase." Fig. 4 illustrates the concept of phase relations of zones of the subgrade with the shaded areas representing the "in phase" zones. The spacing between the centers of the shaded zones is determined by the wave length, which, in turn, is established by the velocity of propagation of the elastic wave in the subgrade and the frequency of the load applications.

In an infinite, elastic, isotropic, homogeneous body disturbances may be propagated by waves of volume change, designated as the compressive wave, push wave, or P-wave, and by waves of distortion of constant volume designated as the shear wave, transverse wave, or S-wave. The compression and shear waves also transmit disturbances throughout the interior of a semi-infinite elastic, isotropic, homogeneous body, but, because of the free surface, a third type of wave appears. This surface wave has been designated as the

Rayleigh wave or R-wave. Additional waves appear in non-isotropic or layered materials and are described, for example, by L. D. Leet (10) and W. M. Ewing, W. S. Jardetzky, and F. Press (11). The velocity of propagation of the shear wave is given by

$$v_s = \sqrt{\frac{G\,g}{\gamma}} = \sqrt{\frac{G}{\rho}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

in which G is the shear modulus of elasticity, γ is the unit weight, and ρ is the mass density of the subgrade material. The velocities of the compression and Rayleigh waves are dependent on Poisson's ratio and are related to the shear wave velocities as shown in Fig. 5.

For actual materials, the different wave velocities may be determined by seismic methods in the field or in the laboratory. The significant variables which affect these velocities are the modulus of elasticity, the unit weight, and Poisson's ratio, as shown by Eq. 8 and Fig. 5. For soils, the water content and confining pressure contribute to changes in these variables and, as a consequence, seasonal variations in water content, for example, will cause seasonal changes in the wave velocities at a particular location. The compressional wave velocity for granular materials varies approximately with the 1/6 power of the confining pressure as shown by the test data assembled in Fig. 6.

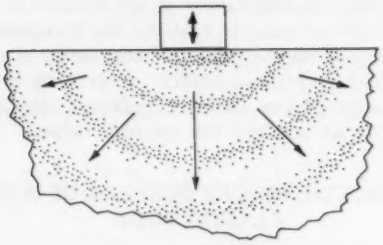


FIG. 4.—PROPAGATION OF ELASTIC WAVES INTO THE SOIL BENEATH AN OSCILLATOR

Vertical Impulses Acting at the Surface.—H. Lamb (17) studied the effects produced by a single impulse which acted at the surface of a semi-infinite, isotropic, homogeneous, elastic solid. He considered primarily the effects produced at the surface and found that the disturbance produced by the impulse spreads over the surface in the form of a symmetrical annular wave system. His theory showed nothing corresponding to the long succession of to and fro vibrations which are characteristic of the seismograph records.

Lamb also studied the effects produced by periodic vertical and horizontal forces applied at a point or distributed along a line on the surface of the semi-infinite solid. By integration of the effects of the periodic vertical point load over a circular area, Reissner obtained (2) a solution which could be used to approximate the behavior of a vibrating mass resting on the surface of a semi-infinite solid. Reissner's solution considered the pressure to be uniformly distributed over the circular base area. Later, Sung (3) and Quinlan (4) studied the effects resulting from uniform, parabolic, and rigid base types of pressure distributions over the circular loading area.

The vertical periodic force which acts on the surface of the semi-infinite solid can be generated by a vibrator having a force output independent of frequency, or by a mechanical vibrator consisting of rotating eccentric weights.

The oscillator shown in Fig. 7(a) develops the same force amplitude Q_1 at all frequencies. The oscillator shown in Fig. 7(b) produces a force depending on the angular velocity of two unbalanced masses m_1. The radius of rotation of the center of gravity of each mass is designated by 1 and the masses rotate at an angular velocity, ω, in opposing directions to balance the horizontal components of the rotating forces. The unbalanced vertical force has an amplitude, Q_1, determined by

$$Q_1 = 2\, m_1\, 1\omega^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

In their theoretical studies of the problem, Reissner, Sung, and Quinlan considered the contact area between the oscillator and semi-infinite body to be

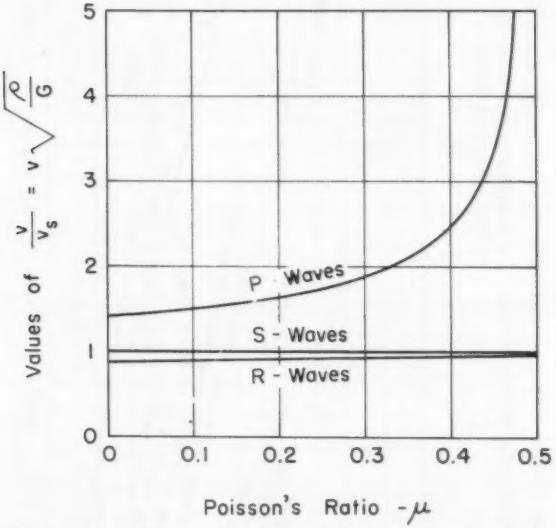


FIG. 5.—RELATION BETWEEN POISSON'S RATIO, μ, AND VELOCITIES OF PROPAGATION OF COMPRESSION (P), SHEAR (S), AND RAYLEIGH (R) WAVES IN A SEMI-INFINITE ELASTIC MEDIUM

circular and of radius r_o. The dynamic characteristics of the vibrator-soil system were found to depend on the parameters: (a) the radius r_o of the loading area, (b) the total mass m_o (= W_o/g) of the oscillator, (c) the amplitude of the dynamic force applied (Q_1), (d) the distribution of the contact pressure on the circular oscillator base, and (e) the Poisson's ratio, μ, mass density, ρ, and shear modulus, G, of the foundation material. A significant term used in the analysis is the dimensionless quantity

$$b = \frac{m_o}{\rho\, r_o^3} = \frac{W_o}{\gamma\, r_o^3} = \frac{\pi\, p_o}{\gamma\, r_o} \quad \ldots\ldots\ldots\ldots (10)$$

which denotes the mass ratio of the system. The quantity b can be interpreted as the ratio between the static mass of the oscillator and the mass of a cylinder of the elastic half-space having a radius r_o and a height r_o/π. Note that

From Duffy and Mindlin (12) - tests on steel balls 1/8 ± 0.000010" dia.

From Gassmann (13) - theoretical - assuming dry spheres of granite, e =0.36.

From MIT (14) - range of test data for Ottawa Standard Sand, e = 0.53.

From Matsukawa and Hunter (15) - test data for dry sand, e = 0.64.

From Shannon, Yamane, and Dietrich (16) - computed from test data for Ottawa Standard Sand, e = 0.61.

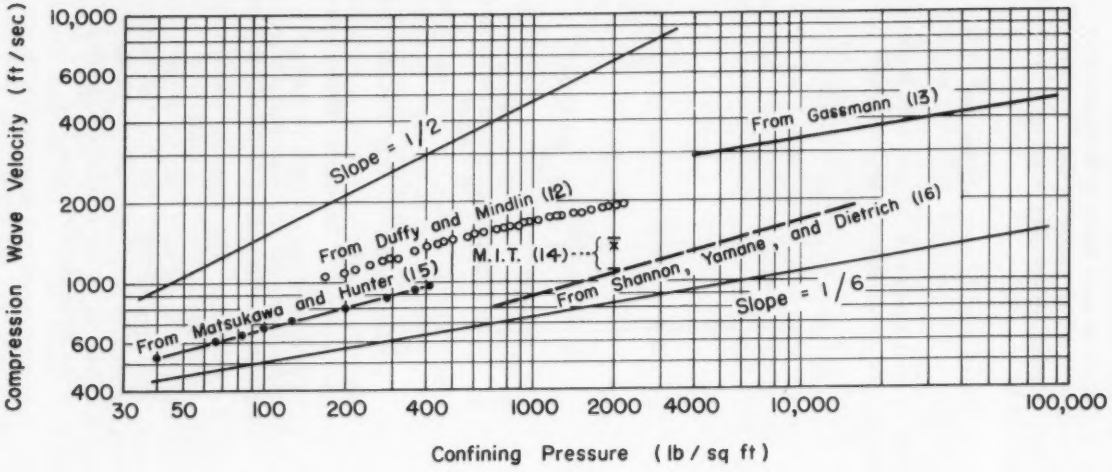
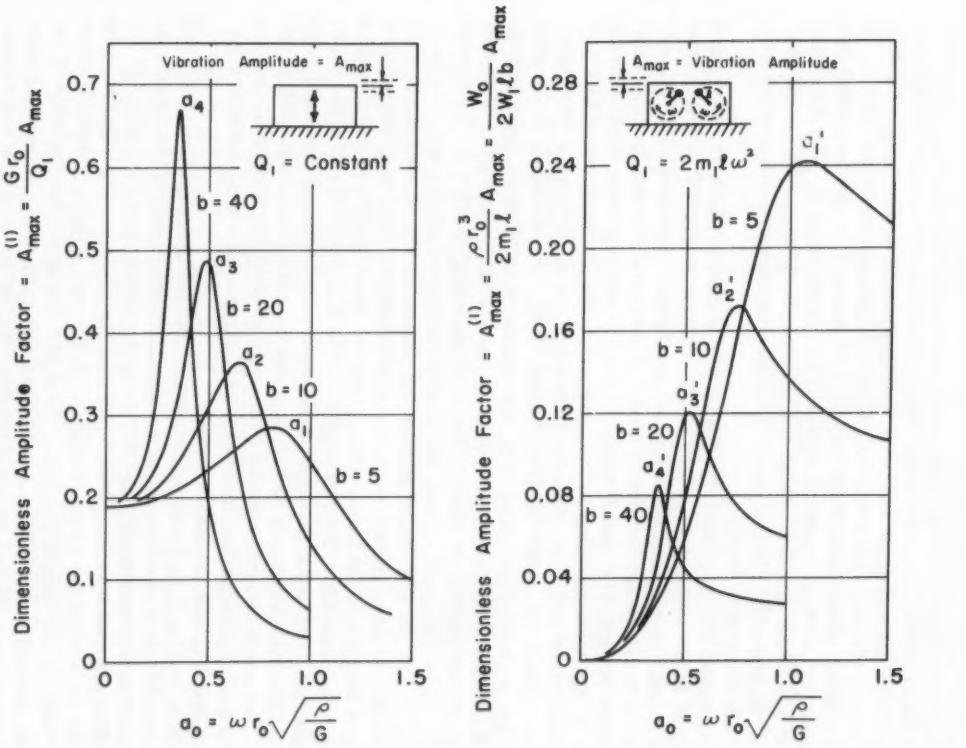


FIG. 6.—EFFECT OF CONFINING PRESSURE ON THE COMPRESSION WAVE VELOCITY IN GRANULAR MATERALS



(a) For Constant Amplitude or Exciting Force

(b) For Exciting Force Amplitude Dependent upon the Exciting Frequency.

FIG. 7.—AMPLITUDE VS. FREQUENCY RELATIONS FOR RIGID BASE OSCILLATOR RESTING ON ELASTIC SEMI-INFINITE BODY ($\mu = 0.25$)

for submerged soils the total saturated density is used in evaluating ρ. A dimensionless frequency term is also utilized which has the form

$$a_o = \omega r_o \sqrt{\frac{\rho}{G}} = 2\pi f_o r_o \sqrt{\frac{\rho}{G}} \quad \ldots\ldots\ldots\ldots (11)$$

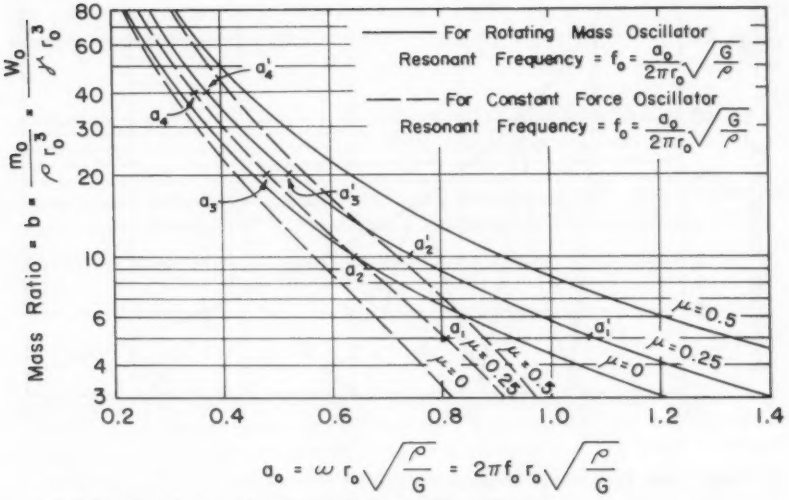
in which f_o is the resonant frequency for the forced vibration.

By using these dimensionless mass ratio and frequency factors along with the elastic characteristics of the subgrade, a theoretical solution for the amplitude of oscillation was obtained. For convenience in further computations, the actual amplitude was converted into a dimensionless amplitude factor. In Fig. 7 the variation of this amplitude factor, with changes in the frequency factor, are shown for several constant values of the mass ratio, b. The diagrams of Fig. 7 result from the assumptions of Poisson's ratio of 0.25 for the subgrade material and a distribution of contact pressure corresponding to that produced by a rigid circular base.

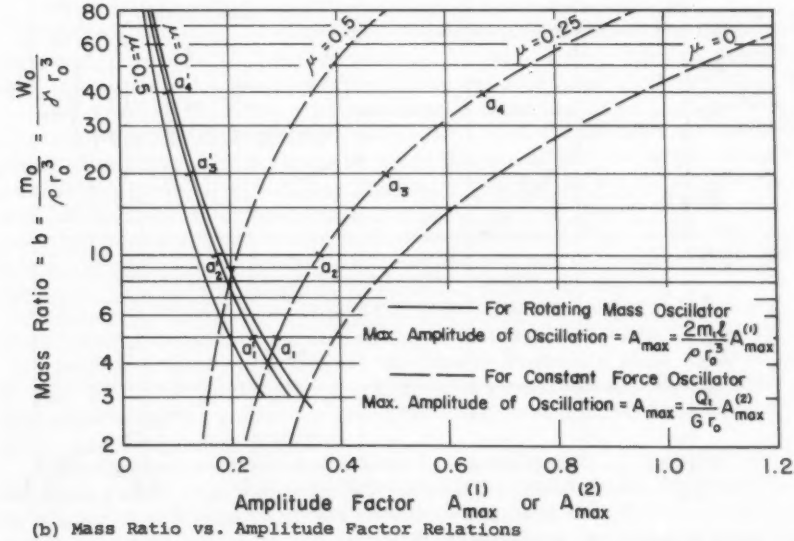
Fig. 7 (a) shows the amplitude-frequency factor curves which are developed by a constant force oscillator, whereas the curves on Fig. 7(b) result from consideration of the rotating mass type of oscillator. The curves in Fig. 7(a) and 7(b) are similar in appearance to those obtained for damped forced vibrations as shown in Figs. 3(a) and 3(b) for the corresponding types of oscillators. The value of a_o corresponding to the peak amplitude for each curve in Fig. 7 may be used to determine the resonant frequency for a particular oscillator-soil system. Thus, by assembling the values of a_o corresponding to the peak amplitude for a particular value of b, the relations between a_o and b may be established as shown in Fig. 8(a). Also, by taking the value of the peak amplitude factor and plotting it against the corresponding value of b, the curves of Fig. 8(b) were obtained. Thus, the points designated a_1, a_2, a_3, ... a'_1, a'_2, a'_3, ... in Fig. 7 determine the points in Fig. 8 indicated by the same symbols. In this manner, Sung has condensed the results of voluminous computations into graphical form, which may be used readily for analysis or design. His diagrams show the relations between b, Poisson's ratio, the pressure distribution, and the maximum power input required, and their influence on the dimensionless frequency factor, a_o, and the maximum amplitude of oscillation. The computations made by Sung were used by the writer to prepare Fig. 8, which is applicable for the condition of a rigid base type pressure distribution on the circular oscillator base.

The curves in Fig. 8 may be used to determine the resonant frequency and maximum amplitude of oscillation for an idealized equivalent of the actual machine foundation. It is necessary that the designer obtain data including (a) the unbalanced forces and operating frequencies of the machines, and (b) at least approximate values for the shear modulus, G, the unit weight, γ, and effective Poisson's ratio of the supporting soil. Then the designer has a choice of the footing-soil contact area (which establishes the radius, r_o, of an equivalent circular area of contact) and the static weight, W_o, of the machine and foundation. These quantities may be varied to provide acceptable dynamic characteristics for the installation. A detailed illustration of the use of the curves in Fig. 8 for the study of a machine foundation is presented subsequently.

Rocking Oscillation.—The rocking type of oscillation of a mass resting on the surface of an elastic semi-infinite body consists of a periodic rotation about a horizontal line passing through the base of the mass (Fig. 9). Arnold, Bycroft, and Warburton (5) have given a solution to this problem for the conditions of a cylindrical mass resting on an elastic semi-infinite body which has

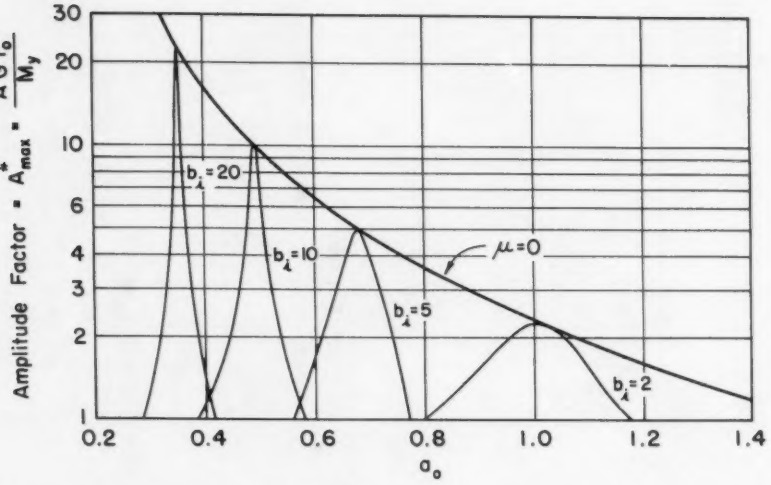

For Rotating Mass Oscillator
Resonant Frequency = $f_o = \frac{a_o}{2\pi r_o}\sqrt{\frac{G}{\rho}}$
For Constant Force Oscillator
Resonant Frequency = $f_o = \frac{a_o}{2\pi r_o}\sqrt{\frac{G}{\rho}}$
Mass Ratio = $b = \frac{m_o}{\rho r_o^3} = \frac{W_o}{\rho r_o^3}$
$a_o = \omega r_o\sqrt{\frac{\rho}{G}} = 2\pi f_o r_o\sqrt{\frac{\rho}{G}}$
$\mu = 0.5$
$\mu = 0.25$
$\mu = 0$

(a) Mass Ratio vs. Frequency Factor Relations

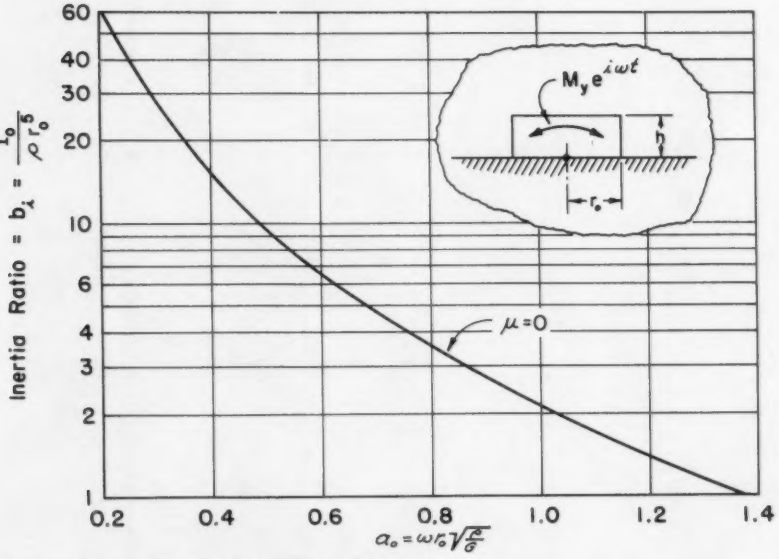

Mass Ratio = $b = \frac{m_o}{\rho r_o^3} = \frac{W_o}{\rho r_o^3}$
$\mu = 0.5$
$\mu = 0.25$
$\mu = 0$
For Rotating Mass Oscillator
Max. Amplitude of Oscillation = $A_{max} = \frac{2m_1 l}{\rho r_o^3} A_{max}^{(1)}$
For Constant Force Oscillator
Max. Amplitude of Oscillation = $A_{max} = \frac{Q_1}{G r_o} A_{max}^{(2)}$
Amplitude Factor $A_{max}^{(1)}$ or $A_{max}^{(2)}$

(b) Mass Ratio vs. Amplitude Factor Relations

FIG. 8.—CHARACTERISTICS OF VERTICAL OSCILLATION FOR AN OSCILLATOR WITH A RIGID CIRCULAR BASE RESTING ON A SEMI-INFINITE ELASTIC MEDIUM



(a) Amplitude Factor vs. Frequency Factor.



(b) Inertia Ratio vs. Frequency Factor.

FIG. 9.—CHARACTERISTICS OF ROCKING OSCILLATION FOR AN OSCILLATOR WITH A RIGID CIRCULAR BASE RESTING ON A SEMI-INFINITE ELASTIC MEDIUM

Poisson's ratio equal to zero. The amplitude of the periodic moment, M_y, was considered to be independent of frequency and the distribution of pressure at the circular base of the oscillator was assumed as the rigid base distribution.

For the rocking condition, the significant parameter is the "inertia ratio," b_i, which is defined by

$$b_i = \frac{I_o}{\rho r_o^5} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which I_o is the mass moment of inertia of the oscillator. For a cylindrical oscillator as shown in Fig. 9, which has a uniform distribution of its mass, the expression for I_o is

$$I_o = \frac{W_o}{g}\left(\frac{r_o^2}{4} + \frac{h^2}{3}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

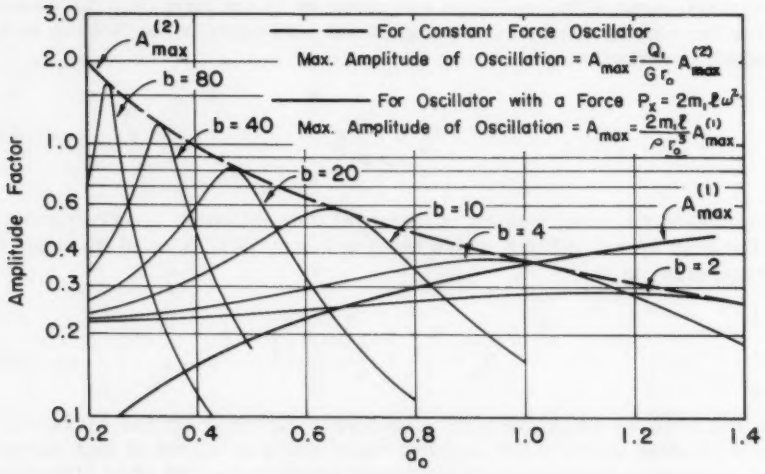
In Fig. 9(a) the amplitude versus frequency curves are shown for b_i = 2, 5, 10, and 20. Also shown is the envelope curve which is tangent to each curve of amplitude versus frequency for the different values of b_i. The point of tangency is close to the point of maximum amplitude, particularly for larger values of b_i, and this tangent curve is used to define the relation between frequency at maximum amplitude and the value of the inertia ratio, b_i, which is shown in Fig. 9(b).

In analyzing a given oscillator-soil combination, the value of b_i may be computed from the known physical constants of the system. Then by using Fig. 9(b), the value of a_o corresponding to this value of b_i is found directly. The resonant frequency can be computed directly from the value of a_o determined previously and the value of amplitude of oscillation at resonance may be determined from Fig. 9(a). It should be noted that the amplitude versus frequency curves for a given value of b_i are much narrower for the rocking case (Fig. 9) than for the case of vertical oscillation (Fig. 7). This means that the range of frequency which will excite resonant or near resonant amplitudes of oscillation is much smaller for the rocking case, and that a relatively small change of frequency will reduce the vibration amplitude to small values.

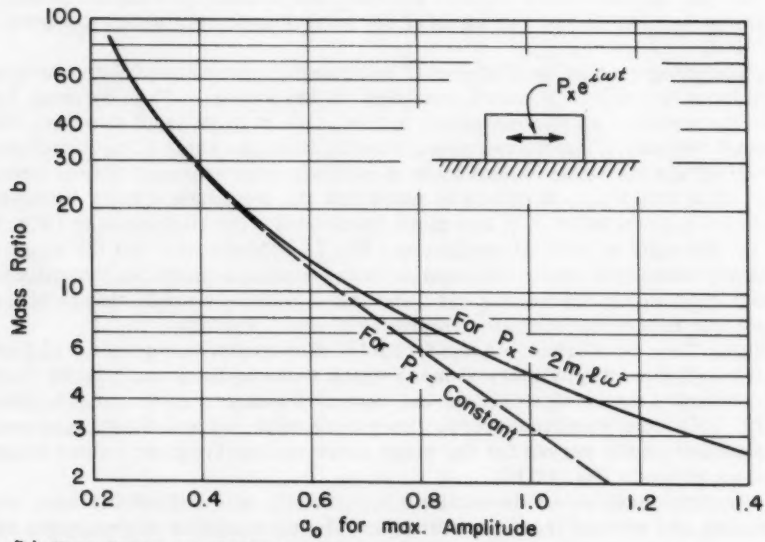
Sliding Type Oscillation.—Arnold, et al (5) also treated the problem of horizontal oscillation of a cylindrical mass which rests on the semi-infinite body. They evaluated the results only for the case of Poisson's ratio equal to zero. In Fig. 10(a) the amplitude versus frequency relations are shown, and from this information the curves for the mass ratio versus frequency were established as shown in Fig. 10(b).

A foundation which tends to oscillate horizontally will undoubtedly have both the sliding and rocking types of oscillation. The possibilities of resonance can be estimated for each condition independently, and then the resulting motion determined by superposition. Superposition of these motions will usually establish a center of rotation of the foundation at some distance into the subgrade.

Interference Effects.—The theoretical solutions consider that only one oscillator is acting on the surface of the semi-infinite body. In many practical cases, several machines are placed within the same building on adjacent foundations. The question becomes: "What is the effect of one machine installation



(a) Amplitude Factor vs. Frequency Factor.



(b) Mass Ratio vs. Frequency Factor.

FIG. 10.—CHARACTERISTICS OF SLIDING OSCILLATION FOR AN OSCILLATOR WITH A RIGID CIRCULAR BASE RESTING ON A SEMI-INFINITE ELASTIC MEDIUM

upon the dynamic behavior of a neighboring installation?" The variables involved in this problem include the weight of the foundation blocks and machinery, the shape and elevation of the block bases, and the dynamic characteristics of the subgrade. In order to study the interference between two rectangular footings, J. W. Clawson (18) mounted two rectangular "feet," each having plan dimensions of 3.5 in. by 11.25 in., beneath a single rotating mass oscillator. The center line spacings of these rectangular feet were varied from 4 in. to 20 in., or from about 1 to 6 footing widths. Preliminary tests, using only dense sand as the subgrade material, indicated that for spacings from 1 to approximately 4 footing widths, the footings behaved as a single unit and their behavior could be treated by use of Fig. 8. For spacings greater than approximately 4 footing widths they acted as independent footings. Further tests using different ratios of length to width for rectangular footings, and different subgrades must be carried out before definite relations can be established for these interference effects.

Effect of a Layered Subgrade.—Vertical oscillations of a mass at the surface of a single elastic layer resting on a rigid medium, were treated by G. B. Warburton (19) (20). He considered the rigid base type of pressure distribution to exist at the circular oscillator base. The additional parameter which becomes significant in the study of oscillators on a layered subgrade is the ratio, R, of the thickness of the layer to the radius, r_o, of the oscillator base.

For different values of b (defined by Eq. 10), it was found that the amplitude of oscillation was many times larger than that occurring for the same value of b and a semi-infinite medium. This is caused by the reflection of the elastic waves from the rigid surface underlying the elastic layer. However, for values of R of approximately 6 or greater (that is, for a layer more than 3 oscillator base diameters in thickness) it was indicated that the dynamic behavior would correspond to that for a semi-infinite medium.

The amplitude-magnifying effect of a rigid surface underlying the elastic layer should not constitute a practical difficulty. If a rigid layer (rock) is within 3 foundation diameters of the design base elevation, it would be logical to support the block directly on piles to rock.

In the intermediate case the subgrade modulus increases with depth below the ground surface. For a subgrade consisting primarily of clean sand, the increase in stiffness with depth will approximate a continuous function. An average value of the elastic soil constants must be estimated for the zone of the subgrade which is effective in supporting the foundation block. Then these values can be used with the theory of homogeneous subgrades (Fig. 8) to evaluate the dynamic characteristics of the foundation. If several distinct layers of marked difference in stiffness exist in the subgrade, the use of the homogeneous subgrade theory could give only a very crude approximation to the correct values.

Oscillator on a Weightless Subgrade.—The similarity between the curves of Figs. 3 and 7 was led to attempts to establish a correlation between the results from the theory of damped forced vibrations and the theory considering the elastic semi-infinite body. Reissner (2) found that a correlation was possible for any specific case, but that a general relation could not be established which would permit extrapolation from one b-value to another. The extrapolation from one value of b corresponding to a model to another value of b corresponding to the prototype is necessary if this kind of procedure is to be of any use in design.

Another approach is to eliminate any attempts to determine a damping constant and to compute the natural frequency of free vibration of a weight supported by a weightless, undamped spring. This procedure will give no information about the amplitude of resonant vibration but will give a value of frequency. It is of interest to evaluate the approximation involved.

For the case of a weight having a rigid circular base resting on the surface of a weightless elastic medium, the spring constant for vertical motion is

$$c_s = \frac{P_z}{z} = \frac{4 G r_o}{(1 - \mu)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

By substituting this value of c_s into Eq. 4, the natural frequency of vertical oscillation is determined as

$$f_n = \frac{1}{\pi} \sqrt{\frac{G r_o g}{W_o (1 - \mu)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

or

$$f_n^2 = \frac{G r_o g}{\pi^2 W_o (1 - \mu)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

In order to evaluate the difference between the natural frequency as given by Eq. 15 and the resonant frequency given by the theory which includes the subgrade weight (as in Fig. 7), R. Jones (21) noted that the parameters b and a_o (Eqs. 10 and 11) contain many of the terms included in Eq. 16. That is,

$$b a_o^2 = \frac{W_o}{\gamma r_o^3} \frac{4 \pi^2 f_o^2 r_o^2 \gamma}{g G} = \frac{4 \pi^2 f_o^2 W_o}{G r_o g} \quad \ldots\ldots\ldots (17)$$

or

$$f_o^2 = \frac{G r_o g}{\pi^2 W_o} \frac{b a_o^2}{4} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Eqs. 16 and 18 will be equal when

$$\frac{b a_o^2}{4} = \frac{1}{1 - \mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

The relation given in Eq. 19 was used to prepare the curves shown in Fig. 11, which show the approximations involved in the use of the weightless subgrade procedure.

The solid curves in Fig. 11 result from excitation provided by a rotating mass oscillator and the dashed curves result from excitation by a constant

force oscillator. Previous comparisons made by Reissner (2), Arnold et al (5), and Jones (21), using diagrams similar to Fig. 11 have considered only the constant-force oscillator and curves corresponding to the dashed curves in Fig. 11 have been obtained. Consequently, it is a little surprising to find the close agreement between the weightless-subgrade results and those obtained from consideration of the rotating mass oscillator. Differences of less than 10% are unimportant when compared to the probable errors in estimating the elastic constants for the subgrade.

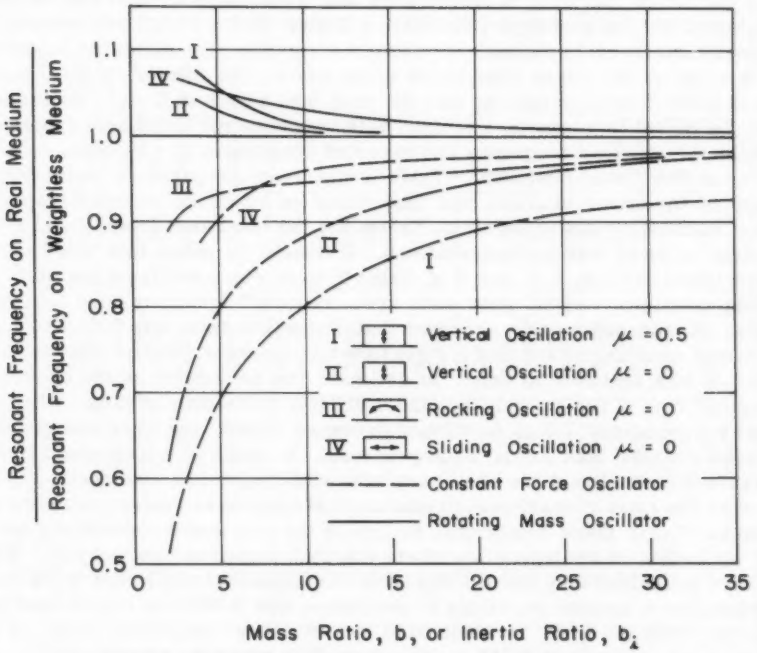


FIG. 11.—COMPARISON OF THE RESONANT FREQUENCY OBTAINED BY CONSIDERING THE SOIL AS AN ELASTIC SEMI-INFINITE BODY HAVING WEIGHT TO THAT OBTAINED BY CONSIDERING THE MEDIUM TO BE WEIGHTLESS (FOR A RIGID BASE TYPE OF PRESSURE DISTRIBUTION ON THE CIRCULAR OSCILLATOR BASE)

When the foundation has a base shape which can be approximated by a circle, the dynamic characteristics of the installation can be determined directly by use of Fig. 8. If the foundation plan deviates markedly from the circular shape, or if the foundation is embedded to an appreciable depth, the subgrade can be treated as a weightless spring for computation of the natural frequency. Then, from the computed value of the mass ratio, b, the probable deviations from the correct resonant frequency can be estimated with the aid of Fig. 11.

VALUES FOR THE SHEAR MODULUS OF SOILS

For actual soils the shear modulus depends on the magnitude of the static and dynamic loads as well as on the water content, which may have a seasonal

variation near the surface. Consequently, it is often difficult to determine a G-value for a particular soil which is actually a constant. However, a range of G-values can usually be established which permits a reasonable estimate of the foundation frequency and amplitude of vibration.

The preferable methods of estimating the soil constants involve the use of data obtained at the actual site. These data may be obtained by evaluating the dynamic behavior of a small oscillator or they may be obtained by seismic methods. If borings show the subgrade to be reasonably uniform, then the use of an oscillator operating at frequencies and amplitudes comparable to those anticipated for the prototype foundation will give useful design information. If the oscillator test data include the resonant frequency plus either the amplitude of vibration or the power input to the drive motor, the method of determining both G and π described by Sung may be used. The values of G and μ determined from the model test may then be used with the prototype foundation data to determine the resonant frequency and expected amplitudes of vibration.

Use of Oscillator Test Data.—Table 1 has been prepared to show typical values of the shear modulus and amplitudes of vibration obtained from test data of oscillators acting on soils. Except for the two cases from Toriumi (6) the data refer to vertical oscillations. It should be noted that most of the values listed in Cols. 6, 7, and 9 of Table 1 were computed from test data including oscillator weight, base plate area, resonant frequency, and eccentric weight. It was arbitrarily assumed that Poisson's ratio was 0.25 (but 0 for horizontal oscillation) and that a rigid base type pressure distribution existed.

Col. 9 was included in order to estimate the probability of the oscillator "jumping" free of the ground when the oscillator force acts upward. When the quantity $A\omega^2$ equals 1.0 g, no contact pressure exists, and when this quantity becomes greater than 1.0 g, a jump occurs. It would be anticipated that the pressure distribution at the oscillator base would approach a parabolic distribution as the ratio of maximum to minimum average base contact pressure increases. Under these conditions, failure of the soil would undoubtedly occur near the edges of the base plate where the confining pressure was low. This condition is indicated by test No. 206 from the DEGEBO tests listed in Table 1, for which the computed amplitude of oscillation was 0.0064 in. (rigid-base assumption, with $\mu = 0.25$) as contrasted to a measured amplitude of 0.0208 in. A computed amplitude of 0.015 in. is obtained by using the assumptions of $\mu = 0.25$ and a parabolic base pressure distribution in conjunction with the appropriate curves given by Sung (3). For the higher vibration amplitudes, the oscillator acts as a compactor and develops a harder "lump" of sand directly below the center of the oscillator (22), which tends to change the pressure distribution at the oscillator base. Test data given by A. Pauw (22) and the writer (24) have pointed out that it is very likely that the base pressure distribution changes as the vibration amplitude becomes more violent.

From a careful consideration of the data given in Col. 9 of Table 1 and the brief examination in the preceding paragraph, it should be evident that oscillator test data can be quite misleading for design purposes if the test conditions are not comparable to the design conditions. The right hand boundary of the limiting curve from E. Rausch (8) which is shown in Fig. 2 corresponds to values of $A\omega^2$ equal to 0.50 g. Thus at the maximum upward position of the foundation during oscillation, the soil pressure is reduced to one-half the static value, and at the maximum downward position the soil pressure is 1.5 times the static value. K. Terzaghi has pointed out (25) the variations in the "hysteresis" modulus of soils which would be anticipated for differences in the range of the

TABLE 1.—OSCILLATOR TEST DATA

Material	Test No.	Oscillator Base Area, in sq ft	Static Contact Pressure in psf	Resonant Frequency, in cps	Vibration Amplitude, in in.	Computed Value of G in psi	Assumed Poisson's Ratio	A ω^2	Type of Oscillator	Motion	Reference in Bibliography
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)[a]	(11)[b]	(12)
Berlin sand, e = 0.53	52	2.69	1,110	21.9	0.041[c]	2,360	0.25	2.01 g	R	V	(1)
	54	2.69	1,110	22.8	0.020[c]	2,560	0.25	1.09 g			
	56	2.69	1,110	27.5	0.007[c]	3,720	0.25	0.77 g			
Micaceous fine sand	209	10.76	315	29.5	0.008[c]	2,330	0.25	0.67 g			
Loamy sand	206	10.76	315	23.2	0.0064[c] 0.0208[d]	1,430	0.25	1.15 g			
Dry silty clay	A	0.175	430	108		4,520	0.50		E	V	(21)
		0.175	920	77		4,660	0.50				
		0.175	1,520	59		4,550	0.50				
Wet, soft silty clay	B	0.175	430	58		1,300	0.50				
		0.175	920	44.5		1,560	0.50				
		0.175	1,520	38		1,890	0.50				
Wet, soft silty clay	C	0.785	73	110		2,270	0.50				
		0.785	430	50		2,000	0.50				
	C	0.136	410	77.5		1,800	0.50				
		0.136	2,470	41		3,040	0.50				
Dry, soft silty clay	D	0.349	216	96		2,570	0.50				
		0.349	760	52		2,400	0.50				
	D	0.087	825	71.5		2,410	0.50				
		0.087	3,014	43		3,010	0.50				
Clean dense quartz sand		0.136	420	46.8	0.0022[c]	1,830	0.25	0.49 g	R	V	(18)
		0.136	660	39	0.0018[c]	1,990	0.25	0.28 g			
		0.136	960	33	0.0013[c]	2,060	0.25	0.14 g			
	1	686	1,230	3.75	0.0077[c] 0.0059[d]	6,700	0		R	H	(6)
	3	576	1,080	4.17	0.0091[c] 0.0075[d]	3,400	0				

[a] R denotes rotating mass, E denotes electromagnetic. [b] V denotes vertical motion, H is horizontal. [c] Computed, using Fig. 8 or Fig. 11. [d] Measured.

maximum to minimum stresses. This factor should be considered in planning the oscillator tests.

G. Values from Elastic Wave Velocities.—Evaluation of the soil parameters can also be estimated from tests to determine the velocities of the elastic waves through the soil. Generally, this is carried out at or near the ground surface and it is necessary to evaluate the wave velocities for the zone of soil which will be effective in supporting the foundation. The theories, procedures, and equipment involved in the determinations of elastic wave velocities in soils have been considered by R. K. Bernhard and J. Finelli (26) and Bernhard (27). Test data obtained by Jones (21) at sites A and B permitted evaluation of the shear modulus at the two sites as 5,750 psi and 1,870 psi, respectively. These values compare favorably with the values listed in Table 1, which were obtained by oscillator tests at the same sites. Toriumi (6) also used the elastic-wave data to evaluate the shear modulus at the sites indicated as tests 1 and 3 in Table 1. By this method he obtained G = 6,060 psi for test 1 and G = 3,400 psi for test 3, values which compare well with 6,700 psi and 3,400 psi as shown in Table 1.

The velocity of propagation of elastic waves varies with the type of soil and its water content. For granular soils, the velocity of propagation of the longitudinal wave also varies approximately to proportion to the 1/6 power of the confining pressure. The curves shown in Fig. 6 indicate the relations between the compression wave velocity and confining pressure for several series of tests on granular materials. It would be anticipated that the shear wave velocity would vary in a corresponding manner. Test data obtained by K. Iida (28) (29) have also shown a marked change in both the compression and shear wave velocities in granular materials with an increase in water content. However, these tests were run on small vertical cylinders confined only by a thin cellophane shell, with the result that the confining pressure at any point in the cylinder was caused only by the weight of the overlying material. Probably the effect of water content in drastically reducing the shear modulus would become of less importance as the confining pressure is increased.

The use of elastic wave velocities for evaluation of soil properties at specific locations is becoming a more common practise in recent years. In interpretation and use of the data it is necessary to recognize, as is also the case for concrete, that the elastic modulus determined from the wave velocities is likely to be higher than that which will occur under the operating conditions. This increase exists because only very small amplitudes of motion occur during the wave propagation whereas motions of a higher order of magnitude are induced by a vibrating foundation. The use of both a small oscillator and the elastic wave procedure should permit determination of soil constants which would "bracket" the effective values.

USE OF PILES

For high-speed machinery having operating speeds greater than 1,000 rpm, it is usually satisfactory to provide a foundation which has a natural frequency of one-half to one-third of the operating frequency. During starting and stopping, the machine will operate briefly at the resonant frequency of the foundation. Thus, the designer must estimate the probable amplitudes of vibration both at resonance and at the operating frequency, which may be done with the aid of Figs. 3 and 8.

At the other extreme, heavy, low-speed machinery requires a foundation which has a natural frequency of at least twice the operating speed of the machine. When the foundation rests directly on the soil the natural frequency of the unit may be increased by reducing the total static weight of the assembly, by increasing the base area of the foundation, or by increasing the G-value of the soil by compaction or by chemical injection. In certain installations, adjustment of these variables within the allowable limits will not increase the frequency to a satisfactory value. Often the use of piles will provide the required stiffness.

When point-bearing piles to rock are installed, an estimate of the possible increase in stiffness of the foundation due to the pile support may be obtained through the theory of longitudinal elastic wave propagation through bars. Even when the pile is unloaded a longitudinal exciting force will cause the pile to vibrate longitudinally at its natural frequency. This phenomenon has been well understood by engineers concerned with pile driving, and studies of this problem have been presented by A. E. Cummings (30), and E. A. Smith (31), among others. A weight added to the upper end of the pile lowers the natural frequency of longitudinal oscillation. From the theory given by S. P. Timoshenko (32), Fig. 12 was prepared to show the influence of pile length and working stress on its natural frequency under the conditions of absolute fixity at the rock surface. Note that the theoretical value is the maximum frequency that could be expected from the use of a point bearing pile. The actual frequency would be somewhat lower because the point would cause an elastic compression of the rock at the point of contact. Fig. 12 shows quickly whether an increase in resonant frequency of the foundation can be provided economically by the use of point bearing piles which carry a negligible load by skin friction.

USE OF THE THEORETICAL PROCEDURES

To demonstrate the use of some of the theoretical methods described previously, two typical block type foundations for machines are analyzed. Before the dynamic behavior of the foundation can be treated, it is necessary to evaluate the unbalanced forces which exist in the particular type of machine. The manufacturer's data may state the unbalanced forces explicitly, or it may be necessary to compute these forces from the weights and dimensions of the moving parts. W. K. Newcomb (33) has presented a description of the unbalanced forces in multicylinder engines which may be used, or it may be preferable to refer to the basic principles of forces developed in crank-type engines as given, for instance, by J. P. Den Hartog (34).

In addition, an evaluation of reasonable values for the shear modulus, G, and Poisson's ratio, μ, of the soil must be made by any one of the procedures indicated previously.

Example A - Foundation for a Single-Cylinder Engine.—Fig. 13 shows a hollow concrete foundation block which was designed to rest on the subgrade just below the basement level and to support the engine at the first-floor level. The subgrade is a sandy clay for which the soil constants were estimated to be G = 2,500 psi, $\mu = 0.50$, and $\gamma = 95$ lb per cu ft.

The single-cylinder engine operates at 1,800 rpm, at which speed the vertical primary unbalanced force is 3,450 lb and the secondary unbalanced force

MATERIAL	E (psi)	γ (lb/ft^2)
Steel	29.4×10^6	480
Concrete	3.0×10^6	150
Wood	1.2×10^6	40

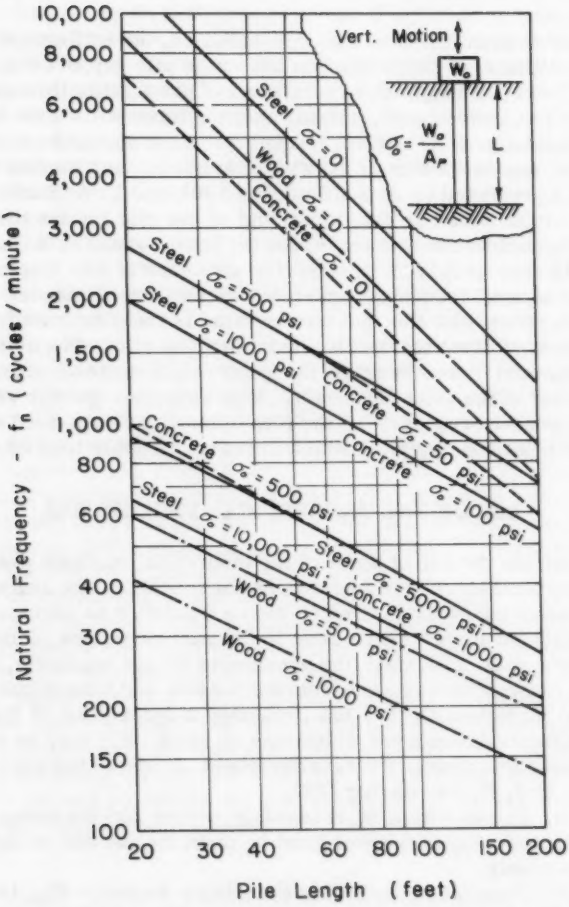
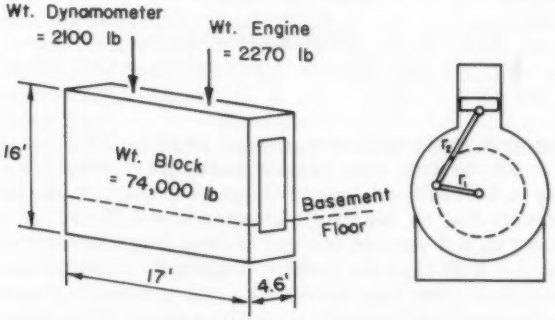


FIG. 12.—RESONANT FREQUENCY OF VERTICAL OSCILLATION FOR A POINT-BEARING PILE CARRYING A STATIC LOAD W_o. (LOADED STRATUM IS RIGID)

is 1,075 lb. The expression for the acceleration of the piston

$$\frac{d^2 x_p}{dt^2} = r_1 \omega^2 \left(\cos \omega t + \frac{r_1}{r_2} \cos 2\omega t \right) \quad \ldots\ldots\ldots (20)$$

establishes the fact that the primary unbalanced force pulsates at the same frequency as the crankshaft rotation, whereas the secondary unbalanced force oscillates at twice the crankshaft frequency. The terms r_1 and r_2 in Eq. 20 refer to the crank throw and connecting rod length, respectively, as shown in Fig. 13.



Single Cylinder Engine

Bore	= 5.125	in.
Stroke	= 6.5	in.
r_1	= 3.25	in.
r_2	=10.75	in.
Speed	= 1800	rpm
Total Wt.	= 2270	lb.
Reciprocating Weight	= 11.87	lb.

Unbalanced Forces

Primary	= 3450 lb. at 1800 rpm
Secondary	= 1075 lb. at 1800 rpm

FIG. 13.—MACHINE AND FOUNDATION DATA FOR EXAMPLE A

The design conditions for this installation require that the resonant frequency of vertical oscillation be less than 900 rpm (half the operating frequency) and that the amplitude of vibration be less than ± 0.0005 in.

For the block without any footing extension, the static soil pressure is 0.5 tons per sq ft, which is less than one quarter of the estimated allowable bearing pressure for the sandy clay. The pulsating vertical motion caused by the unbalanced engine forces may be considered in two parts, one a vertical load applied directly above the centroid of the foundation base, and a moment which tends to cause a pitching motion of the block. For a first approximation to the vertical vibration characteristics of the foundation block, consider a circular

footing having a radius r_o determined by

$$r_o = \sqrt{\frac{\text{base area}}{\pi}} = \sqrt{\frac{78.2}{\pi}} = 5.0 \text{ ft}$$

Then the mass ratio b is

$$b = \frac{W_o}{\gamma r_o^3} = \frac{78{,}370}{95 \times 125} = 6.6$$

From Fig. 8(a), $a_o = 1.14$, and the resonant frequency is

$$f_o = \frac{a_o}{2\pi r_o}\sqrt{\frac{G g}{\gamma}} = \frac{1.14}{10\pi}\sqrt{\frac{2500 \times 144 \times 32.17}{95}} = 12.7 \text{ cps or } 760 \text{ rpm}$$

Note that for this example G and μ were given as 2,500 psi and 0.50, respectively. In case the test data are not sufficient to establish a value of μ, but require that μ be assumed before G can be evaluated, the frequency and amplitude corresponding to the range between $\mu = 0.25$ and $\mu = 0.50$ must be evaluated. From Fig. 8 it is seen that the highest frequency, for a given value of b, occurs for $\mu = 0.50$, and the greatest amplitude of oscillation occurs for $\mu = 0$. For materials other than hard rock, μ is generally greater than 0.20, with the result that soils which would normally support block-type foundations exhibit effective Poisson's ratios between 0.25 and 0.50, for conditions involving small strains.

Because the rotating speed of the machine will pass through the resonant frequency during starting and stopping, it is necessary to evaluate the amplitude of vibration at 760 rpm as well as for the operating speed of 1,800 rpm. The greatest vertical force occurs in each cycle when the primary and secondary maximum forces act together, which for this example produces a total force at 760 rpm of

$$F_{760} = 807 \text{ lb} = 2 m_1 l \omega^2$$

and

$$2 m_1 l = 0.127 \text{ lb sec}^2$$

From Fig. 8(b), $A^{(1)}_{max} = 0.170$ for $b = 6.6$, or

$$A_{max,760} = \frac{2 m_1 l}{\rho r_o^3} A^{(1)}_{max.} = \frac{0.127 \times 0.170 \times 32.17}{95 \times 125} = 0.000058 \text{ ft or } 0.00070 \text{ in.}$$

The operating speed of 1,800 rpm is 2.37 times the resonant frequency of 760 rpm. Consequently, the use of Figs. 3 and 11 will permit an estimate of the amplitude of vibration expected at 1,800 rpm. The actual resonant frequency

will be approximately 1.07 times that predicted by the weightless subgrade theory, which fives $f_1/f_n = 2.37 \times 1.07 = 2.52$. For this frequency ratio the quantity $N_1 (f_1/f_n)^2$ is 1.19 and with the foundation and machine data of W_o = 78,370 lb, W = 11.87 lb, and 2 m_1 l = 0.127 lb sec^2, the computed amplitude of vibration at 1,800 rpm is

$$x_{max} = A_{1800} = 1.19 \frac{0.127 \times 386}{78,370} = 0.00074 \text{ in.}$$

The value of A_{1800} should be less than $A_{max,760}$ but the computed value is slightly higher in this case because of the approximate method used.

Because both values of amplitude are higher than the design limit, but the frequency is satisfactory, it is necessary to increase the effective circular radius, r_o, without decreasing the value of the parameter b appreciably. This means that sufficient weight must be added to maintain the ratio b approximately constant as r_o is increased. If the foundation base width is increased to 6.0 ft, without changing the length, the new radius of the equivalent circular base area is r_{o_2} = 5.7 ft. The ratio of new to old radius, cubed, is

$$\left(\frac{r_{o_2}}{r_o}\right)^3 = \left(\frac{5.7}{5.0}\right)^3 = 1.48$$

Consequently, to maintain the value of b constant, the total static weight of the foundation block must be increased by the factor 1.48; from 78,370 lb to 116.000 lb. The values of amplitude now become, $A_{max,760}$ = 0.00047, and x_{max} = A_{1800} = 0.00050 in., which correspond to the design requirements. If a factor of safety is required, the base area and weight could be increased still more; the size of the factor of safety would depend on the accuracy of determination of the soil and machine constants and the consequences resulting from larger vibration amplitudes.

The unbalanced periodic moment tends to cause pitching of the block in the direction of the long axis. Because of the great stability of the foundation in that direction, this motion should be insignificant. Both the pitching and rocking oscillations of the foundation may be studied by the procedure given in Example B.

Example B - Foundation for a Horizontal Piston-Type Compressor.—To study the dynamic behavior of a foundation for a low-speed machine, consider the installation of a motor-compressor unit on soil having the same characteristics as that for Example A. The air compressors consist of horizontally opposed cylinders; large unbalanced horizontal forces are produced but the vertical unbalanced forces are negligible. The important characteristics of the machine and foundation unit are shown in Fig. 14.

The mass moment of inertia of the foundation block and machinery, about a line through the center of the base, is

$$I_o = \frac{W_o}{3g}\left(h^2 + \frac{a^2}{2}\right) + \frac{W_{mach}}{g}(14)^2 = 2.92 \times 10^6 \text{ ft-lb sec}^2$$

A uniform circular cylinder, 10 ft high, which would have the same value of I_o, would have a radius of $r_o = 13.4$ ft. For this equivalent body

$$b_i = \frac{I_o}{\rho r_o^5} = \frac{2.92 \times 10^6 \times 32.17}{95 \times (13.4)^5} = 2.3$$

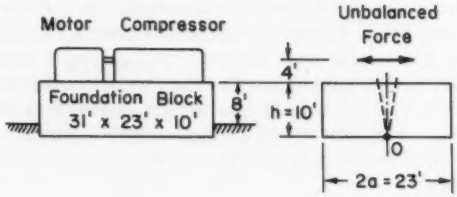
From Fig. 10, $a_o = 0.97$, $A^*_{max} = 2.5$, and the first estimate of frequency is

$$f_o = \frac{a_o}{2\pi r_o}\sqrt{\frac{G}{\rho}} = 4.02 \text{ cps or } 240 \text{ rpm}$$

The amplitude of rotational oscillation at this frequency is

$$A_{240} = \frac{A^*_{max} M_y}{G r_o^3} = \frac{2.5 \times 14 \times 5730 \left(\frac{240}{277}\right)^2}{2500 \times 144 \times (13.4)^3} = 0.00017 \text{ rad.}$$

or at a distance of 14 ft above the assumed point of rotation, the horizontal amplitude of vibration is 0.030 in.



WEIGHTS

Motor	=	40,000 lb.
Compressor	=	156,000 lb.
Foundation Block	=	1,000,000 lb.

MACHINE CHARACTERISTICS

4000 H.P. at 277 rpm
Unbalanced Forces:
Primary = 4680 lb.at 277 rpm
Secondary = 1050 lb.at 277 rpm

FIG. 14.—MACHINE AND FOUNDATION DATA FOR EXAMPLE B

These values of resonant frequency and amplitude of oscillation were obtained using the curves from Fig. 9, which were obtained on the basis of assuming Poisson's ratio equal to zero. From a consideration of Fig. 8, it would be expected that for a value of $\mu = 0.5$ that the resonant frequency would be

higher and the amplitude of vibration would be lower than the value computed on the basis of $\mu = 0$.

As a check on the resonant frequency value of 240 rpm the constant for the soil, considered as a weightless spring, may be determined and the natural frequency computed from Eq. 4. From these computations it was found that $f_n = 290$ cpm when $\mu = 0$ and $f_n = 410$ cpm when $\mu = 0.5$. Thus the natural frequency was increased by a factor of 1.4 by changing μ from 0 to 0.5.

Because the operating frequency for the machine is 277 rpm, the resonant frequency of the foundation should be at least two times the frequency of the secondary force (554 rpm). Consequently, the block as shown in Fig. 14 does not represent a foundation with satisfactory dynamic characteristics. The best way to improve the dynamic behavior of this foundation is to reduce the height of the machinery above the base. If this cannot be done, a reduction in the weight of the block and spreading of the footing would help some, but it would be more advantageous to stiffen the support. If rock is within a reasonable distance below the base elevation, point-bearing piles may be used. From Fig. 12 the materials, lengths, and working stresses can be selected which will provide a natural frequency of 1,100 cpm or greater. If rock is beyond the reach of bearing piles, friction piles may be used to transfer the foundation load to the more rigid underlying soil. In this case care must be taken during construction to insure that additional settlements of the friction piles do not occur when the vibratory operating loads are applied. C. F. Dodge and W. F. Swiger (35) have shown the importance of these additional settlements of friction piles caused by vibratory loads.

CONCLUSIONS

From a consideration of the usual damped-forced vibration theory, and the theory for an oscillator resting on the surface of an elastic subgrade having weight, it was demonstrated that the value of resonant frequency was markedly higher when an oscillator had a force amplitude output increasing with rotational frequency than when the oscillator force amplitude was constant. The difference in frequency can be quite significant for design purposes. It was also shown that the natural frequency computed for an oscillator resting on a weightless subgrade was significantly higher than the frequency determined for a subgrade, having weight, which was excited by a constant-force oscillator. However, the resonant frequency for a rotating mass oscillator acting on the actual subgrade was very close to the corresponding value of natural frequency computed for a weightless subgrade. The weightless-subgrade procedure gives no value for the amplitude of vibration at the natural frequency. The theory for the elastic medium having weight must be used for amplitude determinations. These conclusions regarding resonant frequencies and amplitude of oscillation apply to vertical, sliding, and rocking modes of oscillation of the foundation.

Some of the test results previously obtained from oscillators resting directly on soil have indicated a non-linear behavior. This has caused concern about the usefulness of elastic theories for predicting the dynamic behavior of foundations on soil. However, from computed and measured amplitudes of motion of test oscillators, it was found that the dynamic soil pressures were a much higher proportion of the static soil pressure than could be tolerated in a machine foundation. Some of the test oscillators must have even jumped free of

the ground each cycle, thereby acting as a compactor. A change in pressure distribution because of the increase in amplitude of vibration could account for much of the non-linear behavior found in some tests. This emphasizes that oscillator test data may be misleading if the vibration parameters are quite different from the prototype design values.

Oscillator test data as well as seismic methods have been used successfully to establish soil constants at particular locations. The use of these experimentally determined soil constants with the theory has given reasonably good predictions for the dynamic behavior of the prototype foundation. Data were also given to show typical values of the shear modulus for both clays and granular soils. Test data for velocities of the compression wave in granular soils show that the compression modulus of elasticity varies approximately with the 1/3 power of the confining pressure. The shear modulus should vary in a similar manner.

An analysis of two example machine foundations has shown that the theoretical methods gave reasonable results for the resonant frequencies and amplitudes of vibration of these block type foundations. The theoretical procedures demonstrate the importance of varying each of the parameters which affect the dynamic characteristics of a particular foundation. A diagram was also included to show the effectiveness of point bearing piles to rock in stiffening a foundation and thereby increasing its natural frequency.

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APPENDIX I. —NOTATION

The following symbols have been adopted for use in the paper and are presented herewith for the guidance of discussers:

A^*_{max}, A_{max} = amplitude of oscillation (maximum);

$A^{(1)}_{max}$ = dimensionless amplitude factor corresponding to amplitude produced by a rotating mass oscillator;

$A^{(2)}_{max}$ = dimensionless amplitude factor corresponding to amplitude produced by a constant force oscillator;

$\bar{A}$ = area of foundation base;

A_p = area of pile cross section;

$2a$ = width of a strip footing or width of a rectangular base;

a_o = dimensionless frequency factor, or velocity ratio, defined by Eq. 11;

b = mass ratio for vertical or sliding oscillation, defined by Eq. 10;

b' = mass ratio for vertical oscillation of an infinite strip footing;

b_i = inertia ratio for rocking oscillation, defined by Eq. 12;

c_s = spring constant of subgrade, or coefficient of vertical subgrade reaction;

f_o = resonant frequency for forced vibration;

f_1 = frequency of the exciting force;

f_n = natural frequency of free vibration;

G = shear modulus of elasticity;

g = acceleration of gravity;

h = height of the foundation block;

I_o = rotational mass moment of inertia;

l = radius from center of gravity of rotating part to center of rotation;

M_y = external moment causing rocking of the foundation;

m_o = total static mass of oscillator = W_o/g;

m'_o = static mass per unit of length of infinite strip oscillator;

m_1 = unbalanced rotating mass = W_1/g;

N_1 = displacement magnification factor;

P_x = horizontal force;

P_z = vertical force;

P_o = average soil pressure at the oscillator base;

Q = periodic force;

Q_1 = amplitude of periodic force;

R = ratio of thickness of elastic layer to radius of oscillator base;

r_o = radius of circular base of oscillator;

r_1 = crank throw;

r_2 = connecting rod length;

t = time;

v = velocity of elastic wave;

v_s = velocity of the shear wave;

W_o = total static weight of the oscillator;

W_1 = weight of unbalanced rotating or reciprocating machine part;

x = displacement;

x_s = static displacement, defined by Eq. 3;

x_p = displacement of piston;

z = vertical displacement;

α = phase angle;

γ = unit weight of elastic body or soil;

μ = Poisson's ratio of elastic body or soil;

ρ = mass density of elastic body or soil = γ/g;

σ_o = nominal compression stress in pile caused by static load; and

ω = angular velocity,

λ = damping coefficient;

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PILE-DRIVING ANALYSIS BY THE WAVE EQUATION

By E. A. L. Smith[1]

SYNOPSIS

There are a great many different pile-driving formulas in use, and engineers have never been able to agree as to which one is best. This situation has arisen primarily because, until recently, the mathematics of pile-driving action could not be solved in any practical manner. As a result all pile-driving formulas are partly empirical and, consequently, apply only to certain types or lengths of pile. This paper is presented with the purpose of giving engineers a mathematical method of wider application, depending on the use of electronic computers and numerical integration. The method is also applicable to other impact problems.

INTRODUCTION

Pile-driving formulas are widely used to determine the static bearing capacity of piles. Some of these formulas are also used to determine stresses in the pile during driving.

An astonishing amount of effort and ingenuity has been expended by engineers in the development of pile-driving formulas, with the result that there are a great many different formulas in use, many of which are specified in various building codes throughout the United States and abroad. Robert D. Chellis lists[2] thirty-eight pile driving formulas, and the editors of *Engineering News-Record* have on file four hundred and fifty such formulas.

Note.—Discussion open until January 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. EM 4, August, 1960.

[1] Tucson, Ariz. Formerly Chf. Mech. Engr., Raymond Internatl. Inc., New York, N. Y.

[2] "Pile Foundations," by Robert D. Chellis, McGraw-Hill Book Co., New York, 1951, pp. 28-33, 449-450, 525-538.

The fact that engineers have been unable to agree on any one pile-driving formula is understandable because, until recently, the mathematics of pile-driving action could not be solved in any practical manner. Pile driving is not a simple problem of impact that may be solved directly by Newton's laws.[3] Pile driving is a problem in longitudinal wave transmission that is covered in a general way by the wave equation, which is well known to mathematicians.

Furthermore, pile driving involves many complications such as the use of capblocks, pile caps, cushion blocks, composite piles, and tapered piles, as well as the elastic-plastic action of the ground and other problems in soil mechanics. As a result of these difficulties, all pile-driving formulas are partly empirical and consequently apply only to certain types or lengths of pile. This paper is presented with the purpose of giving engineers a mathematical method of wider application.

So far as the writer is aware, D. V. Isaacs, in 1931, was the first to point out[4] that wave action occurred during the driving of piles. In 1938, E. N. Fox published[5] a solution of the wave equation applied to pile driving, but, as no electronic computers were available at that time, he was forced to use a number of simplifying assumptions that lessened the value of his solution. At the present time (1960), by using the conceptions of the wave equation and resorting to numerical integration and electronic computers, a solution of the pile-driving problem can be obtained that produces mathematical accuracy within about 5%. This degree of accuracy is more than sufficient in view of our present imperfect knowledge of the physical conditions involved.

The mathematical method described herein may, with slight modification, be applied to other impact problems such as the design of a foundation for a forging hammer, or a fendering system for a dock.

OUTLINE OF NUMERICAL METHOD

The hammer, pile, and other parts involved, such as the capblock and pile cap, are represented as a series of weights and springs as shown in Fig. 1, and the time during which the action occurs is divided into small time intervals such as 1/4,000 sec. The action of each weight and each spring is then calculated separately in each and every time interval. In this way a mathematical determination may be made of stresses, and of pile penetration or permanent set per blow, against any amount or kind of ground resistance.

The process may be compared to making drawings for an animated motion picture. The artists who prepare such drawings must take account of the fact that the film will be projected at 24 frames per sec. Each drawing must therefore differ from the preceding drawings by 1/24 sec. In order that the picture may appear realistic when projected, computations must be made to determine how far any moving object will progress in each 1/24 sec. If the motion is uniform the displacements in each succeeding drawing should be uniform; if the motion is uniformly accelerated, as in the case of a falling object, the displacements should differ by increasing amounts readily calculated by using the well-known laws of uniformly accelerated motion.

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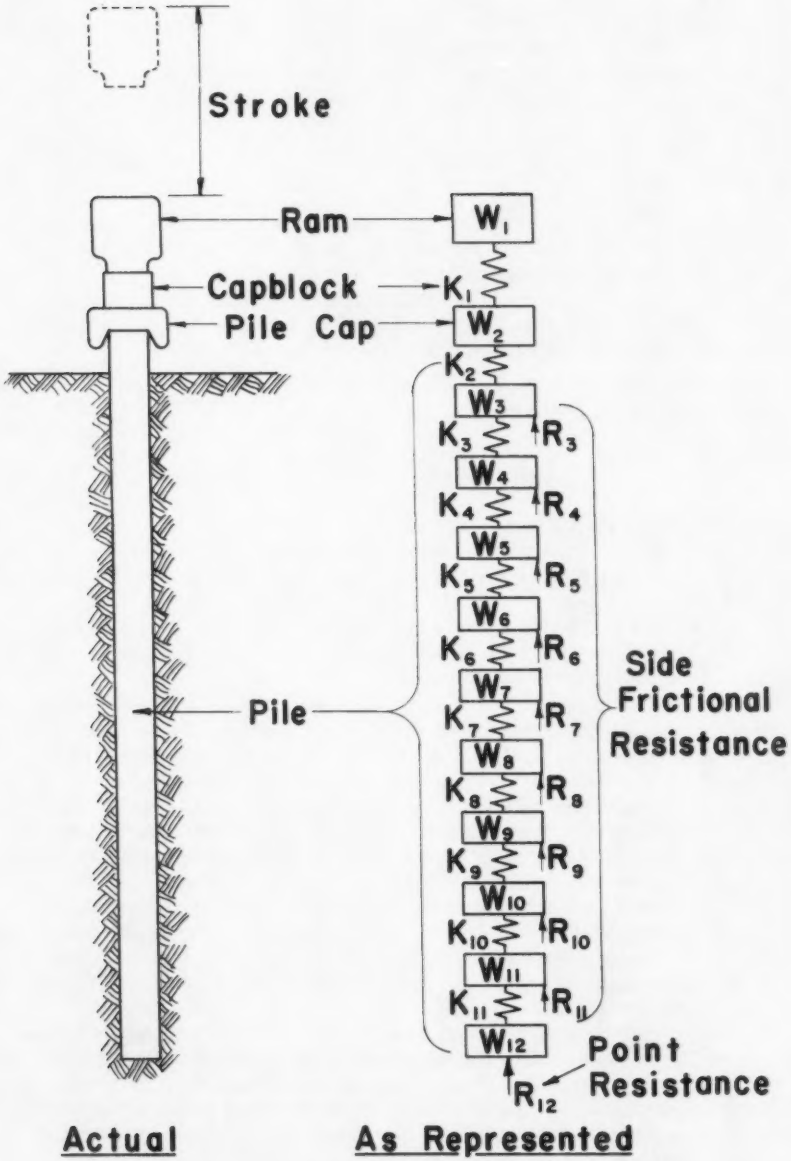

FIG. 1.—METHOD OF REPRESENTING PILE FOR PURPOSE OF CALCULATION

Obviously the rules for making such a set of motion-picture drawings are these:

1. Time is divided into intervals of 1/24 sec each.
2. No motion can be shown in any one drawing.
3. Motion is depicted by making each successive drawing differ from the preceding drawing by just enough to represent the changes occurring during one interval.

The rules for making a numerical pile calculation are almost identical, and may be stated thus:

a. Time is divided into small intervals such as 1/4,000 sec.
b. It is assumed that all velocities, forces, and displacements will have fixed values during any particular interval.
c. The velocities, forces, and displacements for each interval will be computed so as to differ from those existing in the preceding interval by just enough to represent the change occurring during one interval.

If these two sets of rules are compared rule by rule, it will be found that there is no essential difference.

DIAGRAMMATIC REPRESENTATION

The hammer-ram and the pile can are usually short, heavy, rigid objects which may be represented as individual weights without elasticity. In Fig. 1 the first weight W_1 represents the ram, and the second weight W_2 represents the pile cap.

The capblock is a short springy object of wood, plastic, or similar material which is comparatively light and which may therefore be represented by a spring. In Fig. 1 this spring is numbered K_1.

The pile is a heavy object, but is somewhat compressible because of its length. It is therefore subject to wave action under the blow of the hammer. This wave action can be analyzed mathematically by dividing the pile into short sections or "unit lengths" such as 5 ft or 10 ft in length. The weight of each unit length is represented by an individual weight (W_3 to W_{12} in Fig. 1) and the elasticity of each unit length is represented by an individual spring (K_2 to K_{11} in Fig. 1). The motion of each weight and each spring is then calculated as though each were actually a separate and distinct object.

If the pile is of uniform section, the weights and springs representing it are all alike. If the pile is tapered or is a composite pile, or is in any way irregular in cross section, then these weights and springs differ from one another so as to represent approximately the actual distribution of weight and elasticity throughout the length of the pile.

One may wonder at the fact that comparatively large unit lengths, such as 5 ft or 10 ft, give desired accuracy in computing the action of the impact wave. The following analogy, using water waves, may make the matter clear. Of course impact waves in a pile are longitudinal waves, whereas water waves are transverse waves; nevertheless, water waves can be used to illustrate the principles involved:

Imagine a body of water on which moderately long waves are traveling from right to left. If a long strip of flexible material such as sponge rubber is allowed to float on the surface, it will follow the wave action exactly. If for purposes of mathematical analysis we were to represent this flexible strip by a

number of short rigid floats connected together by flexible links, the picture would appear as shown in Fig. 2. This would be an approximation, but the approximation would involve negligible error because the small floats would ride the waves almost exactly like the flexible strip.

But suppose we make the rigid floats comparatively long so that they approach or exceed the length of the water wave, then the picture would look like Fig. 3. These long floats cannot follow the wave form closely. If the floats were made longer still, the resemblance to the true wave form would disappear completely.

From the foregoing, it may be concluded that in dividing a pile into unit lengths for purposes of calculation, the unit lengths chosen must be considerably shorter than the wave length of the stress or impact wave produced by the hammer. Fortunately, a pile-driving impact usually produces a fairly long wave form, therefore division of the pile into lengths of the order of 5 ft to 10 ft will produce acceptable accuracy.

In the special and unusual case where the exact form taken by the impact wave is being investigated, a smaller unit length such as 2 ft or even 1 ft may be advisable. In this case, it may be advisable to divide the ram into unit lengths as well as the pile. In this way a high degree of accuracy may be obtained.

FIG. 2.—SHORT FLOATS

It has been explained previously, that the pile must be divided into unit lengths and that time must be divided into small intervals such as 1/4,000 sec. There is a very important relationship between these two procedures. The smaller the unit lengths chosen, the smaller must be the time interval.

This can be explained in a general way by again considering a motion picture. If the action to be represented is of the normal kind, a 1/24-sec time interval between pictures is suitable. If the action is high speed in character, such as the rotation of a wheel, a much smaller interval is required to show the action accurately. If a 1/24-sec interval is used, the wheel may actually appear to run backward. The same sort of phenomenon occurs in a numerical wave equation calculation because the calculation is a step-by-step process which must keep ahead of the stress wave caused by the hammer blow. The smaller the unit length is made, the smaller must be the time interval. It is also true that especially large or suddenly applied external forces or resistances may require the use of a smaller time interval.

It follows from the foregoing, that whenever a numerical wave equation calculation is made, care must be taken to make certain that the time interval used is not too large. On the other hand, the time interval should not be unnecessarily small, because this would use an unnecessarily large amount of time in computation with little or no increase in accuracy. Consequently, the choice of the correct time interval becomes an important consideration. For

pile calculations, a time interval of about 1/4,000 sec will usually be found to be satisfactory. This matter is discussed further in the Appendix.

GENERAL

The foregoing discussion outlines the basic method for obtaining a numerical solution of the pile-driving problem. This method has the outstanding advantage of dividing the problem into a number of simpler problems each of which may be considered more or less independently.

Before going into the mathematics, let us first consider the practical or physical aspects of the problem.

Soil Mechanics.—In order to make a pile calculation, it must be assumed that the soil will act in some particular way. When future investigators develop new facts, the mathematical method explained herein can be modified readily to take account of them, but on the basis of information presently available, the assumptions listed in what follows are recommended.

Resistance At Point Of Pile.—Chellis gives[2] a precedent to follow, which teaches that the ground compresses elastically for a certain distance, (which

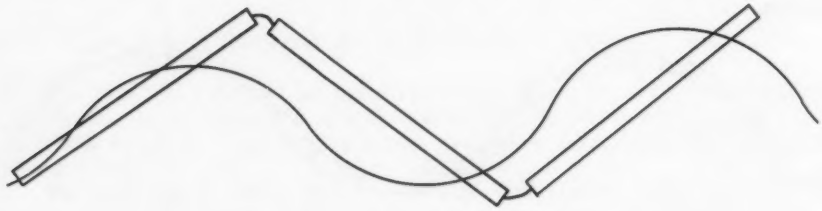

FIG. 3.—LONG FLOATS

Chellis calls C_3, but which herein will be termed Q for "quake") and then fails plastically with constant or "ultimate" resistance R_u. This concept is illustrated in Fig. 4.

Starting at 0, the pile point moves ahead a distance Q (usually assumed to be 0.1 in.) compressing the soil elastically so that at point A the ground resistance has built up to its ultimate value R_u. Plastic failure then occurs and ground resistance remains equal to R_u until the pile point reaches B. Elastic rebound equal to Q then occurs, and motion ceases at point C where all forces are zero. The permanent set of the pile is the distance $s = \overline{OC} = \overline{AB}$.

This conception fails to consider the element of time. Some piles penetrate the ground more rapidly than others. Obviously, the ground will offer more instantaneous resistance to rapid motion than to slow motion. We therefore introduce the additional factor of "viscous damping," which is commonly used in vibration problems.

The numerical wave equation calculation gives the instantaneous velocity of the point of the pile in any time interval. If this instantaneous velocity is called v_p, and if J is a damping constant, then the product $J\,v_p$ may be used to increase (or decrease) the ground resistance so as to produce damping. Thus at any point x on the line OABC of Fig. 4, the instantaneous damping resistance is $J\,v_p\,R_x$. If J is assumed to have a value of 0.15, and if at point x the calculated value of v_p is 2 fps, the damping resistance is $0.15 \times 2 \times R_x = 0.30\,R_x$, and the total instantaneous resistance is $R_x + 0.30\,R_x = 1.30\,R_x$.

The damping resistance is, of course, temporary or instantaneous, and does not contribute to the bearing capacity of the pile. It should also be noted that the constant J refers only to resistance at the point of the pile, such as R_{12} of Fig. 1.

Thus we see that resistance at the point of the pile is calculated to take account of the following:

1. Elastic ground compression or quake "Q."
2. Ultimate ground resistance "R_u."
3. Viscous damping based on a damping constant "J."

Resistance Alongside The Pile.—Resistance alongside the pile is calculated like resistance at the point of the pile, except that a damping constant J' is used instead of the damping constant J which has already been discussed in connection with resistance at the point. Thus in Fig. 1 J' would apply to re-

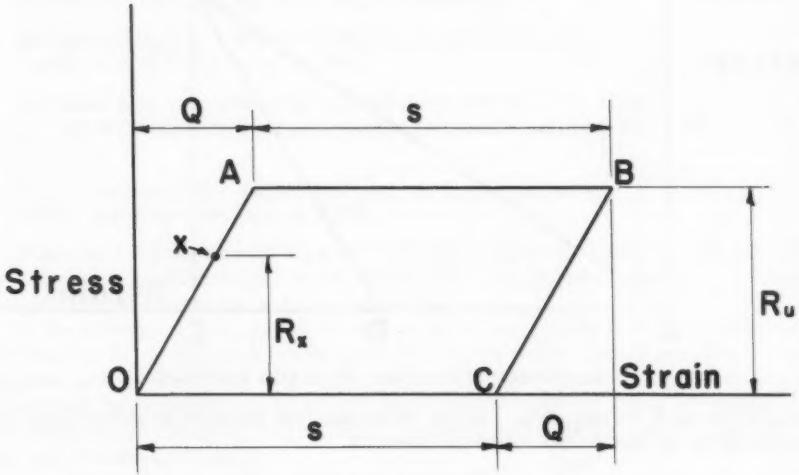

FIG. 4.—STRESS-STRAIN DIAGRAM AT PILE POINT

sistances R_3 to R_{11} inclusive, whereas J would apply only to the point resistance R_{12}.

As the pile is driven downward, the soil under the point of the pile is displaced or caused to flow aside very rapidly. However, the soil alongside the pile is not correspondingly displaced. The value of J' should therefore be smaller than the value of J. A value of J' = 0.05 will be assumed. (This compares with a value of 0.15 assumed for J).

In the numerical wave equation calculation the ground resistance may be distributed over the full length of the pile in any way that is desirable. For simplicity all resistance may be considered to be concentrated at the point of the pile, but the foundation engineer is at liberty to adopt any distribution that in his judgment is best suited to the specific ground conditions as disclosed by borings or other soil investigations.

Physics.—In addition to the action of the soil, we must consider the physical characteristics of the hammer, capblock, pile cap or follower, and pile. Precast concrete piles also require a cushion on the head of the pile. The physical

characteristics of this cushion must also be considered. Each of these elements will now be discussed separately.

Hammer.—The hammer ram is ordinarily a short, heavy, rigid object that can be represented by a single weight without elasticity, such as W_1 of Fig. 1. In special cases where the ram is comparatively long and slender, or where an especially accurate stress analysis is required, it may be advisable to divide the ram into a number of weights and springs (as shown in Fig. 10(b) to be presented subsequently).

The velocity of the ram at the instant of impact is needed in order to start the numerical calculation. Ordinarily the rated foot pounds of energy of the hammer is given by the manufacturer. The efficiency is sometimes given and

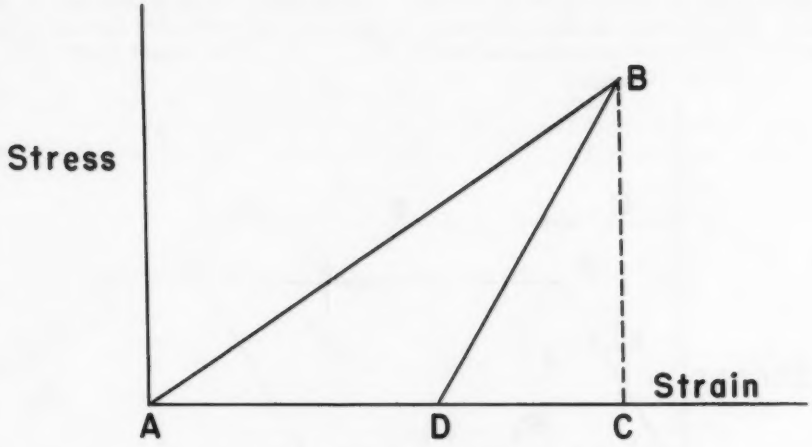

FIG. 5.—STRESS-STRAIN DIAGRAM FOR CAPBLOCK

sometimes must be assumed. From these data the velocity at impact may be calculated by means of the following formula:

$$\text{Velocity At Impact, in fps} = \sqrt{\frac{\text{Rated energy, in ft - lb} \times \text{Efficiency} \times 64.4}{\text{Weight, in lb}}} \quad \ldots (1)$$

Capblock (Termed "Dolly" in England).—The capblock is represented by spring K_1 in Fig. 1. The form of the stress-strain diagram (or the hysteresis loop) that is produced as the capblock is suddenly compressed and then allowed to reexpand, from information now available may be assumed to be as shown in Fig. 5.

Compression occurs along line AB whose slope is determined by the elastic constant K_1 of the capblock. Restitution occurs first along the line BD and then, because the capblock cannot transmit tension, is completed along line DA, thus forming the hysteresis loop ABDA. The slope of the line BD is automatically determined by the electronic computer so that

$$\frac{\text{Area BCD}}{\text{Area ABC}} = \left(e_1\right)^2 = \frac{\text{Energy output}}{\text{Energy input}} \quad \ldots (2)$$

where e_1 is the coefficient of restitution of the capblock. This is in accord with Newton's laws of impact.

Few tests have been made to determine the elastic characteristics of capblocks under impact conditions. The writer's former employer recently conducted a small number of tests by placing capblocks between two horizontally swinging rams weighing 4,800 lb each. The capblock was struck by one ram with about 15,000 ft-lb of energy, and the subsequent motion of both rams was recorded. A lead pellet was so mounted that it was squashed by the blow and thus measured the maximum compression of the capblock. From these measurements, the spring constant and the coefficient of restitution were determined mathematically.

It was found that the characteristics of a wood capblock vary during driving, but the tests led to the conclusion that in order to be on the conservative side in computing pile penetration per blow, a hardwood capblock with grain vertical, 6 in. in height originally, and with a horizontal area of A square inches, may be assumed to have the following characteristics:

Spring constant, K = 20,000 A lb per in. of compression
Coefficient of restitution, e = 50%

The tests also indicated that a Micarta capblock (Nema Grade "C") 12 in. in height has the following characteristics (which do not vary much during driving):

Spring constant, K = 45,000 A lb per in. of compression
Coefficient of restitution, e = 80%

Pile Cap or Follower or Helmet.—Like the ram of the hammer, the pile cap is ordinarily a short heavy rigid object that can be represented by a single weight without elasticity, such as W_2 of Fig. 1.

If the pile cap is long and slender, as is the case when it is to be used as a follower to drive the piles below ground or below water, then it may have to be represented by a number of weights and springs (as shown in Fig. 10(e), to be presented subsequently). In this case elastic constants must be computed. The elastic constants for any object of uniform cross section are computed by the well-known formula:

$$K = \frac{A E}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where A is the sectional area in square inches, E denotes the modulus of elasticity in pounds per square inch, and l is the unit length, in inches, represented by a single spring.

Cushion Blocks (Called "Head Packing" in England).—In Fig. 1, springs K_2 to K11, inclusive, represent the elasticity of the pile itself. However, if a precast concrete pile is being driven, a cushion block must be used under the pile cap W_2 so as to protect the concrete from shattering. In this case, spring K_2 in Fig. 1, would represent the cushion.

Fig. 5 applies to the cushion block as well as to the capblock, and computing methods for the cushion block and capblock are also alike.

Dynamic tests similar to those previously described for capblocks indicate that a wood cushion 4 in. thick composed of pine boards with grain horizontal, as used on top of a precast pile to distribute the blow evenly, may fairly be

assumed to have the following characteristics during driving, where A is the area of the head of the pile:

Spring constant, K = 3,480 A lb per in. of compression
Coefficient of restitution, e = 50%

These characteristics vary widely during driving because the pine boards with grain horizontal become very hard and very elastic under continued pounding. The foregoing values are on the conservative side when used for computing pile penetration per blow.

Pile.—In Fig. 1, the ten springs with elastic constants K_2 to K_{11}, inclusive, represent the elasticity of the pile itself.

The experimental data available indicates small hysteresis loss in the pile itself, consequently, the springs representing the pile elasticity, such as K_2 to K_{11} of Fig. 1, may, with negligible error, be considered to be perfectly elastic. (Eq. 24, which is given in the Appendix, makes possible the inclusion of hysteresis loss in the pile itself if in the future this appears to be worthwhile.)

In Fig. 1, springs K_3 to K_{11} can transmit tension; but springs K_1 and K_2 cannot because the ram, the pile cap, and the pile are separate pieces. Therefore the electronic computer is programmed accordingly.

MATHEMATICS

The foregoing material outlines the problem and discusses the physical conditions that must be taken into account. The following gives the mathematics used for the numerical solution of the wave equation, as applied to pile driving.

In order to set up a program for the electronic computer, a number of equations and routines are required, as will be seen.

Notation.—The subscript m denotes the general case; thus W_m denotes any weight as in Fig. 1. For instance W_m might denote W_5, in which case K_m would denote K_5 and R_m would denote R_5, etc. Thus the letter m denotes position in space. The letter n is used to denote position in time.

The capital letters C, D, F, R, V, and Z, refer to instantaneous values for compression, displacement, force, resistance, velocity, and accelerating force, calculated for any time interval n. The corresponding small letters refer to corresponding values in the preceding time interval n-1. The notation d* refers to a displacement value in interval n-2 (two intervals back). Compressions and displacements are totals up to and including the time interval specified:

A = cross sectional area, in square inches;

C = spring compression in time interval n, in inches;

c = spring compression in time interval n-1, in inches;

D = displacement in time interval n, in inches;

d = displacement in time interval n-1, in inches;

d* = displacement in time interval n-2, in inches;

D' = ground plastic displacement in time interval n, in inches;

d' = ground plastic displacement in time interval n-1, in inches;

E = modulus of elasticity, in pounds per square inch;

e = coefficient of restitution;

F = force exerted by spring in time interval n, in pounds;

g = acceleration due to gravity, in feet per second per second;

J = damping constant applicable to resistance at point of pile (R_{12} of Fig. 1);

J' = damping constant applicable to resistance at side of pile (R_3 to R_{11} of Fig. 1);

K = spring constant, in pounds per inch;

K' = spring constant applicable to ground, in pounds per inch;

l = unit length of pile, in inches;

m = subscript denoting the general case;

n = time interval for which calculations are being made;

p = subscript denoting "at point of pile";

Q = quake or maximum elastic ground deformation, in inches;

R = resistance in time interval n, in pounds;

R_u = total ultimate ground resistance to driving, in pounds;

R_{u_m} = portion of R_u applicable to weight W_m;

s = permanent set of pile per blow, in inches;

T_m = critical time interval for spring m, in seconds;

Δt = time interval used for calculation, in seconds;

V = velocity in time interval n, in feet per second;

v = velocity in time interval n-1, in feet per second;

W = weight, in pounds; and

Z = accelerating force in time interval n, in pounds.

Equations.—In a previous paper[6] the writer derived the following five basic equations from the elementary laws of physics. They are based on the assumption that all springs are perfectly elastic, and that the pile is represented typically as shown in Fig. 1:

$$D_m = d_m + v_m(12\,\Delta t) \quad \ldots\ldots\ldots\ldots\ldots (4)$$

$$C_m = D_m - D_{m+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$F_m = C_m K_m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$Z_m = F_{m-1} - F_m - R_m \quad \ldots\ldots\ldots\ldots\ldots (7)$$

[6] "Impact and Longitudinal Wave Transmission," by E. A. L. Smith, Transactions, ASME, August, 1955.

and

$$V_m = v_m + Z_m \frac{\Delta t\, g}{W_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Referring to Fig. 1, the velocity of any particular weight in any particular time interval, produces a displacement in the next time interval as given by Eq. 4. The displacements of two adjacent weights produce a compression in the spring between them as given by Eq. 5. This spring compression results in a spring force as given by Eq. 6. The two spring forces and the resistance acting on one particular weight produce a net force as given by Eq. 7. This net force accelerates or decelerates the weight and produces a new velocity as given by Eq. 8. This new velocity, in turn, produces a new displacement in the next succeeding time interval, etc. The process is repeated for each weight and each spring in each time interval, until all downward velocity is lost.

Subsequently these equations will be modified so as to provide for inelastic and plastic characteristics of the capblock, cushion block, and the ground.

If, by methods well known to mathematicians,[7] the wave equation, with resistance included, is converted into a difference equation suitable for numerical computation, the following expression is obtained:

$$D_m = 2\, d_m - d^*_m + \frac{12\, g\, \Delta t^2}{W_m}\left[\left(d_{m-1} - d_m\right)K_{m-1} - \left(d_m - d_{m+1}\right)K_m - R_m\right] \quad \ldots (9)$$

Eq. 9 can also be obtained by combining Eqs. 4 through 8 into a single equation. This shows that these five basic equations are equivalent to the wave equation for purposes of numerical computation.

Eqs. 4 through 8 may be combined in numerous ways. For instance, Z_m can be eliminated by combining Eqs. 7 and 8 thus:

$$V_m = v_m + \left(F_{m-1} - F_m - R_m\right)\frac{\Delta t\, g}{W_m} \quad \ldots\ldots\ldots (10)$$

Similarly, C_m can be eliminated by combining Eqs. 5 and 6 thus:

$$F_m = \left(D_m - D_{m+1}\right) K_m \quad \ldots\ldots\ldots\ldots\ldots (11)$$

The ultimate ground resistance R_u, may be distributed throughout the length of the pile in any way desired by writing

$$R_u = R_{u_3} + R_{u_4} + R_{u_5} + \ldots\ldots\ldots\ldots\ldots (12)$$

Any of these individual ultimate resistances may be denoted by the general expression R_{u_m} (Fig. 1).

In connection with Fig. 4, it was explained that the ground is assumed to compress elastically a distance Q, after which the ground fails plastically at constant resistance, which for the general case of weight W_m would be called

[7] "Methods of Applied Mathematics," by F. B. Hildebrand, Prentice-Hall, Inc., New York, 1952, pp. 322-328.

R_{um}. For computation this may be represented diagrammatically as in Fig. 6.

In Fig. 6, springs K'_{m-1} and K'_m are introduced to represent ground elasticity. Spring K'_m, the general case, must compress a distance Q one, in order to reach the individual ultimate ground resistance R_{um}; therefore, to compute the spring constant for the ground we may write:

$$K'_m = \frac{R_{u_m}}{Q} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

In Fig. 6, D_m is the displacement of weight W_m from its original position, as computed by Eq. 4, and D'_m is the measure of the plastic ground failure permitting movement in excess of Q. The amount that ground spring K'_m is compressed may be seen from Fig. 6, to be equal to $D_m - D'_m$. From this we may write an equation for R_m similar to Eq. 11:

$$R_m = \left(D_m - D'_m\right) K'_m \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

In order to include viscous damping as already discussed under the heading "General: Soil Mechanics," Eq. 14 is modified to include a damping constant J or J' multiplied by the instantaneous velocity. For frictional resistance alongside the pile, Eq. 14 thus becomes:

$$R_m = \left(D_m - D'_m\right) K'_m \left(1 + J' v_m\right) \ldots\ldots\ldots\ldots\ldots (15)$$

When applied specifically to resistance at the point of the pile this becomes:

$$R_p = \left(D_p - D'_p\right) K'_p \left(1 + J v_p\right) \ldots\ldots\ldots\ldots\ldots (16)$$

Routines.—Evaluating ground displacements D'_m and D'_p as used in Eqs. 15 and 16 involves the use of computer routines rather than equations.

Routine #1.—The displacement D'_m remains constant (starting at zero) unless changed by one of the two following conditions:

1. D'_m cannot be less than $D_m - Q$
2. D'_m cannot be more than $D_m + Q$

The computer makes these comparisons in each time interval and adjusts the value of D'_m accordingly:

Remarks.—Routine #1 applies only to friction alongside the pile, and insures that the compression or extension of spring K'_m cannot exceed Q, the maximum elastic ground deformation. When the upper part of the pile rebounds after the blow, the motion reverses itself; consequently, routine #1 provides for plastic movement D'_m either in the normal downward direction or in an upward direction resulting from pile rebound. This routine is illustrated in Fig. 7.

Routine #2.—The displacement D'_p remains constant (starting at zero) unless changed by the following condition:

D'_p cannot be less than $D_p - Q$

The computer makes this comparison in each time interval and adjusts the value of D'_p accordingly.

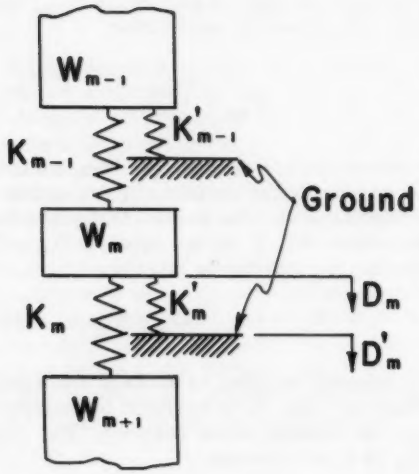

FIG. 6.—GROUND ELASTICITY

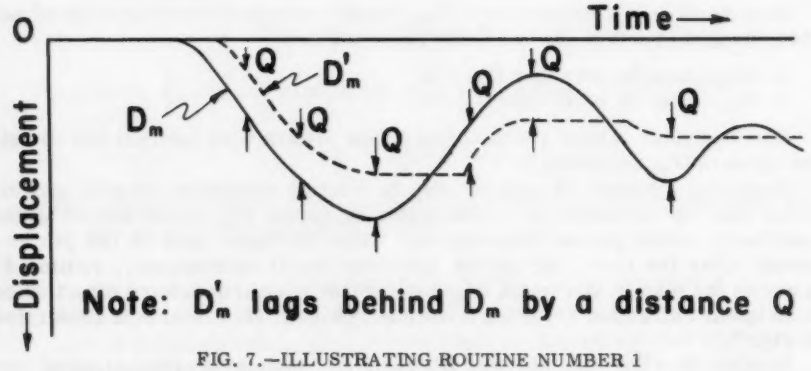

FIG. 7.—ILLUSTRATING ROUTINE NUMBER 1

Remarks.—Routine #2 applies only to the point of the pile. It is similar to routine #1 except that only downward or positive plastic ground movement D'_p is considered. The maximum value of D'_p is the "permanent set" of the pile, which is called s, as shown in Fig. 4. Therefore, it would be illogical to consider reversal of plastic failure of the ground as was done in routine #1. This routine is illustrated in Fig. 8.

Routine #3.—This routine provides for inelasticity in spring K_1 by taking account of its coefficient of restitution e_1, and involves the alternate use of Eqs. 17 and 18 as follows:

$$F_1 = C_1 K_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

for compression. This equation is identified with the number 17 merely as a matter of convenience. Actually it is nothing but Eq. 6 applied to spring K_1. For restitution

$$F_1 = \left[\frac{K_1}{(e_1)^2}\right] C_1 - \left[\frac{1}{(e_1)^2} - 1\right] K_1 C_{1\,max} \quad \ldots\ldots\ldots (18)$$

in which e_1 is the coefficient of restitution for spring K_1 and $C_{1\,max}$ is the maximum value of C_1, used as a constant.

It should be noted that if $e_1 = 1.00$, then Eqs. 17 and 18 become identical and equivalent to Eq. 6.

Eq. 18 is used for restitution. Its derivation is based on the relationships previously explained in connection with Fig. 5, and is not difficult.

Routine #3 is based on a stress-strain (or force-compression) diagram as given in Fig. 9. Letters (a), (b), etc., in Fig. 9, correspond to the correspondingly numbered steps in the routine, as given subsequently.

The actual routine is as follows:

(a) Use Eq. 17 until $C_1 - c_1$ becomes negative. This last value of c_1 is thereafter treated as a constant called C_{1max} for use in Eq. 18.

(b) Use Eq. 18 instead of Eq. 17.

(c) Sometimes recompression occurs; nevertheless, continue using Eq. 18 until $C_1 - C_{1max}$ becomes positive.

(d) Then again use Eq. 17 until $C_1 - c_1$ again becomes negative. This gives a new value for C_{1max} equal to the latest value of c_1.

(e) Then use Eq. 18 with the new value of C_{1max} as a constant. (If additional recompressions occur, steps (c), (d), and (e) are repeated).

(f) F_1 can never be less than zero, that is, can never be negative, because the ram is always a separate piece.

Routine #4.—This routine provides for inelasticity in spring K_2 by taking account of its coefficient of restitution e_2. It uses two equations corresponding to Eqs. 17 and 18, thus:

$$F_2 = C_2 K_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

for compression, and

$$F_2 = \left[\frac{K_2}{(e_2)^2}\right] C_2 - \left[\frac{1}{(e_2)^2} - 1\right] K_2 C_{2max} \quad \ldots\ldots\ldots (20)$$

for restitution.

Note that if the coefficient of restitution $e_2 = 1.00$ Eqs. 19 and 20 become identical and equivalent to Eq. 6.

The routine is the same as routine #3, with subscripts changed from 1 to 2, with the one exception of item (f) of routine 3. For routine #4, item (f) reads as follows:

(f) Alternate No. 1: F_2 cannot be less than zero (that is, cannot be negative).

(f) Alternate No. 2: F_2 can be less than zero (that is, can be negative).

The proper alternate must be specified. Alternate No. 1 applies if K_2 cannot transmit tension. Alternate No. 2 applies if K_2 can transmit tension.

PROGRAM FOR ELECTRONIC DIGITAL COMPUTER

By combining the foregoing formulas and routines a complete program for the computer can be prepared.

The calculation starts with a specified ram velocity V_1 at the "beginning of impact" denoted as time interval 0. All other variables except V_1 start with a value of zero in time interval 0, and remain at zero until modified by one of the following items bearing Roman numerals, which comprise the computer program. These items are listed in the order in which they are calculated by the computer in each and every time interval successively. Time interval #1 is the first time interval for which computations are made by the electronic computer.

Computer Program #1 For Piles With All Resistance At The Point Of The Pile

I Compute D_1 to D_p by means of Eq. 4.

II Compute D'_p by means of routine #2.

III Compute R_p by means of Eq. 16.

IV Compute C_1 to C_{p-1} by means of Eq. 5.

V Compute F_1 by means of routine #3.

VI Compute F_2 by means of routine #4.

VII Compute F_3 to F_{p-1} by means of Eq. 6.

VIII Compute V_1 to V_p by means of Eq. 10.

Computer Program #2 For Piles With Slide Friction As Well As Point Resistance

I to VII inclusive, same as above.

VIII Compute D'_3 to D'_{p-1} by means of routine #1.

IX Compute R_3 to R_{p-1} by means of Eq. 15.

X Compute V_1 to V_p by means of Eq. 10.

In the above programs Eq. 10 has been used instead of Eqs. 7 and 8 because the accelerating force Z is seldom of interest and the use of Eq. 10 saves computer time.

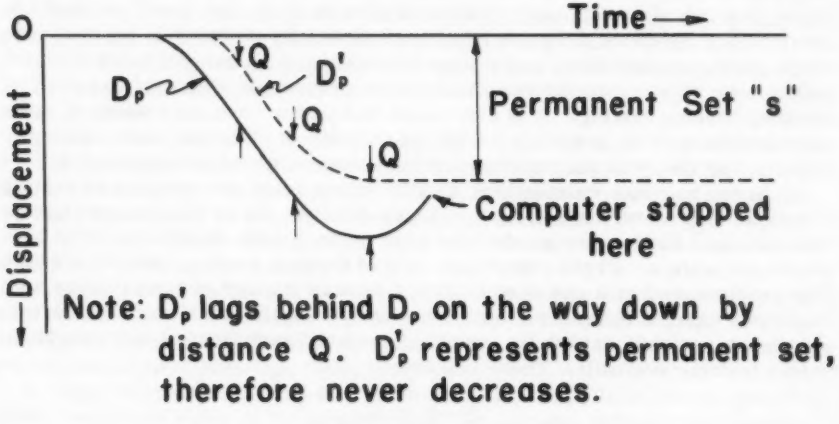

FIG. 8.—ILLUSTRATING ROUTINE NUMBER 2

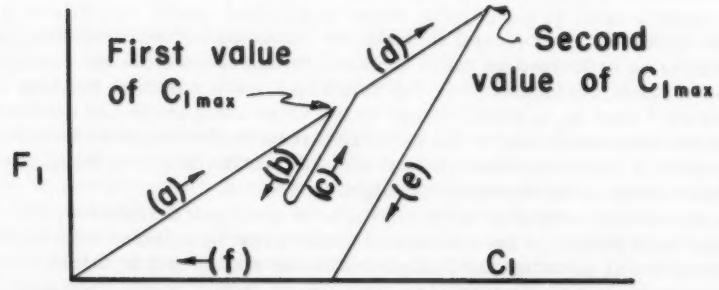

FIG. 9.—STRESS-STRAIN DIAGRAM FOR SPRING K_1

The computer should be programmed to stop automatically when the following two conditions are reached:

(1) $D'_p - d'_p$ equals zero.
(2) V_1 to V_p inclusive, are all simultaneously negative or equal to zero.

This will ordinarily stop the calculation very soon after D'_p reaches a maximum and the point of the pile begins to rebound as indicated in Fig. 8. By this time the major driving force has been expended and only secondary or residual forces are acting. These are of little interest and are difficult to analyze accurately.

RECOMMENDATIONS AND COMMENTS

The mathematical method explained herein was originally developed as a means of computing stresses during driving in capblocks, pile caps, piles, and pile-driving cores or mandrels, in order to insure proper design. Previously all these parts had been designed by trial and error.

During the past 12 yr this mathematical method has been used for over 1,000 calculations, and has been gradually improved and extended to cover pile driving in general. It is now being presented with the hope that it will be useful in determining stresses during driving, and especially stresses in precast concrete piles, so that these piles may be provided with correct reinforcement and correct driving procedures in order to prevent or eliminate cracking or breakage during driving. It is also hoped that in due time the results of many calculations will be correlated with the results of pile-load tests carried to failure, and also with the rapidly increasing knowledge of soil mechanics.

In order to make calculations at all, values must be assigned to certain constants that describe soil action during driving. Up to the present time no instrumented field experiments have been performed to determine these constants accurately. From experience gained through working extensively with this problem, and as a result of making a limited number of comparisons with load tests carried to failure, the writer has personally concluded that certain values are accurate enough for practical use until such time as more accurate values become available. These values are

$$Q = 0.10 \text{ in.}$$
$$J = 0.15$$
$$J' = 0.05$$

The writer is emboldened to make the foregoing recommendations (which may easily be criticized as rash) because he has found from the many calculations already performed that the numerical wave equation solution is not "sensitive," that is, a small change in the value assigned to any constant will produce a smaller change in the calculated results. He therefore believes that the results of calculations are meaningful and worthwhile even though the constants used are not as accurate as might be desired.

To determine a suitable value for R_u to be used in a calculation, the working load to be placed on the pile must be multiplied by a factor such as 2 or 3. However, if soil investigation indicates that the soil is of a type that "sets up" or one that "relaxes" after driving, this factor should be modified accordingly. The numerical wave equation calculation tells what happens during the actual driving of the pile; it cannot predict what will happen a week or a year later. For this information soil mechanics must be consulted.

Modern large electronic computers can complete a calculation such as described in this paper, in a matter of seconds. It is therefore practical to make a large number of calculations and to tabulate (or plot) the results for future reference. Such a tabulation would cover the types of piles and hammers in common use. Special calculations would then need to be made only for unusual types.

The following conclusions come partly from the results of numerical wave equation calculations, but primarily from many years of practical pile-driving experience.

CONCLUSIONS

1. A numerical method suitable for use on modern electronic computers is now available that permits calculation of pile-driving action under any specified set of conditions, and gives permanent set per blow as well as instantaneous stresses, displacements, and velocities.

2. The knowledge of soil mechanics is incomplete, especially the knowledge of soil mechanics under pile-driving action. This offers a fertile field for future investigation, especially now that a mathematical method of calculating driving action is available for checking and analyzing field test results.

3. The knowledge of the physical action of capblocks and cushion blocks under driving conditions could also be improved by dynamic testing. Only a few tests have been made.

4. The numerical wave equation calculation can be used to determine the driving characteristics of various types of piles and hammers. It can also be used to determine the range of application throughout which any particular pile-driving formula may be considered reasonably accurate.

5. It is well known that precast concrete piles are sometimes cracked or broken due to excessive compressive stresses caused by the hammer blow. It is not equally well known that in certain cases excessive tensile stresses may also result from the hammer blow, especially if the pile is long. A wave-equation analysis can be used to determine correct reinforcement and driving procedures to help eliminate these troubles.

6. Many engineers assume that at final penetration, a light pile is easier to drive than a heavy pile of the same overall dimensions. Actually this assumption is correct only for light loading. When loads are heavy and consequently resistance is high, a heavy pile is usually easier to drive than a light pile, because it is stiff and strong and thus is better able to carry large forces down into the ground.

APPENDIX

This appendix is presented primarily for those who actually make calculations, and gives practical details that will be found helpful.

In the writer's paper,[6] a recommendation was made that, in representing the pile, the weight of each unit length should be concentrated at the center. Experience since that paper was written has shown that a preferable procedure is to concentrate the weight of each unit length at the end away from the place where impact occurs, as was done in Fig. 1. This procedure improves accuracy and permits any combination of pile, hammer, etc., to be represented logically as shown in Fig. 10.

The elements shown in Fig. 10 are combined in the following typical ways to form single diagrams similar to Fig. 1:

1. Ram, capblock, pile cap and pile.
2. Ram, capblock and core or mandrel.
3. Ram and pile.
4. Ram, capblock, long follower and pile.
5. Ram, capblock, pile cap, cushion and pile. In this case the spring representing the cushion must be combined with the top spring of the pile by means of Kirchhoff's law

$$\frac{1}{K} = \frac{1}{k_1} + \frac{1}{k_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

6. Long ram, capblock, pile cap and pile. In this case the spring representing the capblock must be combined with the bottom spring of the ram. (See #5 above).

7. Ram, pile cap or anvil and pile. In this case combine the weights representing the ram and the pile cap, and assign a velocity at impact so that the momentum of the combined weight equals the momentum of the ram at impact.

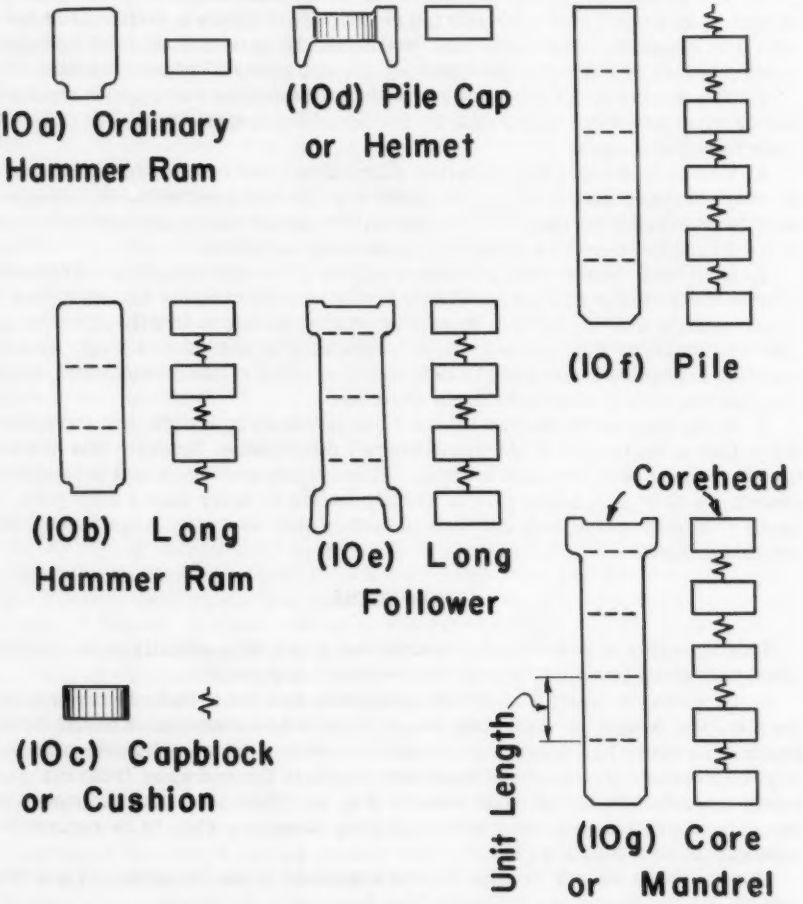

FIG. 10.—REPRESENTATION OF PILE AND DRIVING ELEMENTS

The best time interval Δt, for use in making a pile calculation, may be defined as the largest interval that will produce a completely stable calculation. No simple rule can be given that will cover every possible condition, but the writer has discussed this question and has presented two formulas for the "critical" time interval based on the velocity of travel of the stress-wave as follows: [6]

$$T_m = \frac{1}{1.9648}\sqrt{\frac{W_{m+1}}{K_m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

and

$$T_m = \frac{1}{1.9648}\sqrt{\frac{W_m}{K_m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

The minimum value of T_m that can be obtained by using these formulas is the "critical" one.

The time interval Δt should be about half of this critical value so as to prevent instability from arising due to other factors not included in the above formulas, such as the effects produced by coefficients of restitution e, ground quake Q and damping constants J and J'. As a general rule, the following values for Δt, will be found to be satisfactory for use with piles divided into 8 ft or 10 ft unit lengths:

For steel piles	1/4,000 sec
For wood piles	1/4,000 sec
For concrete piles	1/3,000 sec

If shorter unit lengths are used, the time interval Δt should be reduced proportionately.

An interesting and instructive combination of figures for use with a steel pile of uniform section is to take l = 100 in. (8.33 ft), Δt = 1/4,000 sec, and E = 29,300,000 psi. Sound or stress waves travel at a speed equal to $\sqrt{E/\rho}$ where ρ is the mass per unit volume. Steel weighs 0.283 lb per cu in. and the velocity of sound is 16,660 fps. Thus sound or stress waves will traverse an 8.33 ft unit length in 1/2,000 sec. If 1/4,000 sec is used for Δt, forces will progress from one pile spring to the next (such as from spring K_4 to K_5 of Fig. 1) every second time interval. Similarly velocities will progress from one pile weight to the next (such as W_4 to W_5 of Fig. 1) every second time interval. This may be observed from the numerical wave equation calculation, though the figures will not come out absolutely exact. However, when the calculation has been carried on long enough so that the stress wave is reflected from the point of the pile, the wave action may become harder to observe from the calculated figures, though sometimes the travel of the reflected wave up the pile is easy to follow. If ground friction alongside the pile is included in the calculation, the sound or stress wave will travel at a slower speed.

If the time interval used is too large, instability in the calculation may result. Usually this will force the computer to handle such large numbers that it will give warning. A good plan is to program the computer so that it will give warning if the value of V_2 or V_p, in any time interval, exceeds twice the velocity of the ram at the moment of impact (that is, twice V_1 in time interval 0). This will usually detect instability promptly. It is also a good plan to program the computer to stop the calculation after a maximum of 300 time intervals. This will eliminate any possibility of the computer running indefinitely if the regular program does not stop the calculation at the correct point, as in Fig. 8.

As a check, more than one calculation should be made for any particular pile and hammer combination using varying values of R_u. If four or five such calculations are made, the calculated values of s should plot as a smooth curve. If they do not, it may be concluded that instability was present, or that some error has been made in the calculation. Other methods of checking have been given by the writer.[6]

In rare cases the capblock spring K_1 may be very stiff and the weight W_2 may be very light. This combination tends to produce instability, and if tested according to Eqs. 22 and 23, may call for an excessively small time interval. From a practical standpoint it may be preferable to arbitrarily soften spring K_1 somewhat, or to arbitrarily increase the weight of W_2. Either or both of these changes will permit use of a normal time interval such as 1/4,000 sec, will have only a small effect, and will give results on the safe or conservative side.

It should be noted that if the hammer ram is long and is represented by a number of weights and springs as shown in Fig. 10 (b), then all these weights will have equal initial velocities V_1, V_2, V_3, etc., and the computer must be specially programmed to accept all these velocities as input data.

It should also be noted that the methods described herein may be used to analyze the driving of a pile in which the ram strikes its blow part way down in the pile, or all the way down at the point. Fig. 11 shows, diagrammatically, a pipe pile being driven by a drop hammer operating inside the pile and striking on the point. All equations already given apply to Fig. 11 as well as to Fig. 1 except that the compression of spring K_1 must be computed by the equation

$$C_1 = D_1 - D_{12} \quad \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

the accelerating force for weight W_{12} must be computed by the equation

$$Z_{12} = F_1 + F_{11} - R_{12} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

and the accelerating force for weight W_2 by the equation

$$Z_2 = -F_2 = R_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

These changes are obvious from Fig. 11, and of course require special computer programming.

It is of great interest to note that driving on the point of the pile in this way has been found to be no more effective than driving on the upper end of the pipe. The explanation appears to be that driving on the point of the pile tends to produce high point velocity, which causes high temporary resistance because of soil viscosity. This illustrates the necessity of introducing the damping constant J.

Some types of ground furnish side frictional resistance during driving, but add little or nothing to the permanent bearing capacity of the pile. In this case side resistances may be used in the calculation but ignored in determining the bearing capacity of the pile. Thus in Eq. 12 R_{u_3}, R_{u_4}, etc., would be given values, but only R_{u_p} would be considered to be permanent bearing capacity.

The wrtier has also presented[6] formulas for computing K-values for tapered piles.

It is usual to ignore gravity forces because the pile-bearing capacity wanted is the bearing capacity that the pile has in addition to its own weight. Anyone who wishes to include gravity will find methods of doing so discussed elsewhere by the writer.[6]

Illustrative Problem.—Let Fig. 1 represent a steel pipe or H pile 100 ft long with 15.58 sq in. of area, weighing 53 lb per ft, driven by a #1 Vulcan hammer

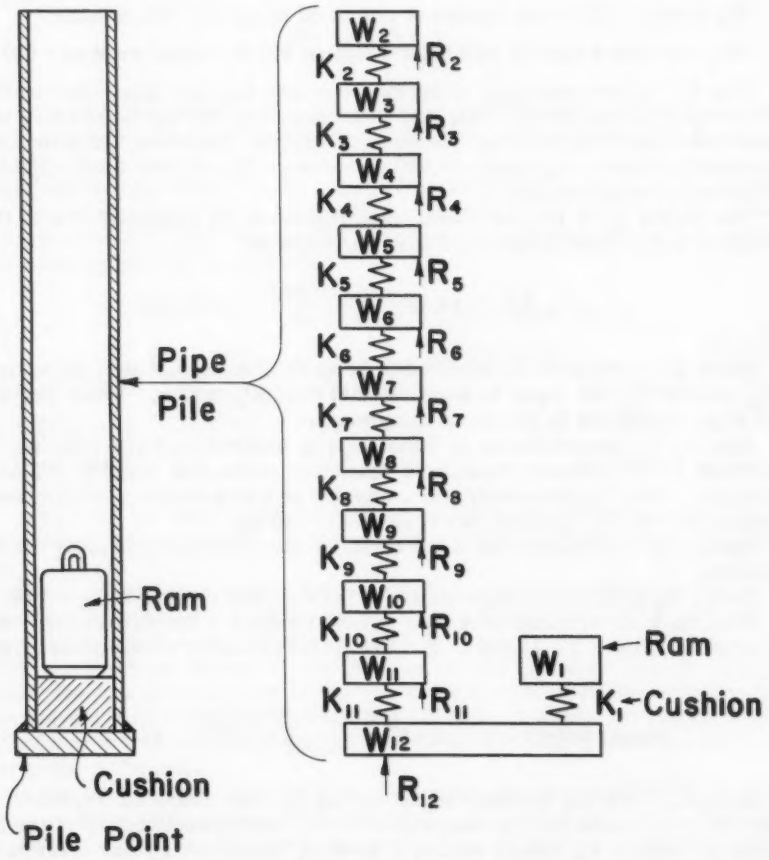

FIG. 11.—PIPE PILE

having a 5,000 lb ram falling 3 ft, using a 700 lb pile cap and a hardwood cap-block 6 in. thick when new and 11-1/2 in. in diameter. A special pile point is specified weighing 100 lb.

Assuming that hammer efficiency is 80%, ground quake Q is 0.1 in., and damping constant J = 0.15, determine the permanent set per blow when driving against an ultimate ground resistance R_u of 200,000 lb concentrated at the point of the pile.

Step No. 1, Choice of Unit Length.—A 10 ft unit length will be adopted. Experience has shown this unit length to be generally satisfactory.

Step No. 2, Choice of Time Interval.—As a general rule, a time interval $\Delta t = 1/4{,}000$ sec will produce a stable calculation using 10 ft unit lengths, and this will be used here.

Step No. 3, Determination of Weights W_m.—

W_1 = the hammer ram = 5,000 lb;

W_2 = the pile cap = 700 lb;

W_3 to W_{11} = 10 ft unit lengths of pile at 53 lb per ft = 530 lb each;

W_{12} includes a special pile point weighing 100 lb, therefore $W_{12} = 630$ lb.

Step No. 4, Determination of Spring Constants K_m and K'_m.—The term K_1 represents the capblock. This has approximately 100 sq in. of area, and it was stated under the heading "General: Physics: Capblock," that the spring constant for such a capblock = 20,000 A, therefore $K_1 = 20{,}000 \times 100 = 2{,}000{,}000$ lb per in. of compression.

The terms K_2 to K_{11} inclusive, each represent the elasticity of a 10 ft unit length of pile. This is computed from the formula:

$$K = \frac{A\,E}{l} = 15.58 \times \frac{30{,}000{,}000}{10 \times 12} = 3{,}900{,}000$$

Since all resistance is assumed to be at the point of the pile, all values of K'_m except K'_p are equal to zero. In this case K'_p is K'_{12}. From Eq. 13 we get $K'_{12} = 2{,}000{,}000$ lb per in. of compression.

Step No. 5, Determination of Coefficient of Restitution.—The pile has been assumed to be perfectly elastic as discussed under the heading "General: Physics: Pile," consequently, the computer program requires coefficients of restitution only for the first two springs K_1, and K_2.

Spring K_1 represents the wood capblock for which a value of $e_1 = 0.50$ is suitable.

Spring K_2 in this case represents part of the pile, therefore, $e_2 = 1.00$.

Step No. 6, Determination of Ram Impact Velocity.—The rated energy of the hammer is $5{,}000 \times 3 = 15{,}000$ ft-lb. The impact velocity is computed by using Eq. 1:

$$\text{Impact velocity, in fps} = \sqrt{\frac{15{,}000 \times .80 \times 64.4}{5{,}000}} = 12.4 \text{ fps.}$$

Step No. 7, Decide Whether or Not Spring K_2 Can Transmit Tension.—The term W_2 represents the pile cap which merely rests loosely on top of the pile, therefore spring K_2 cannot transmit tension, therefore F_2 can never have a negative value and its minimum value will be 0.

Step No. 8, Summary.—The foregoing values may be summarized for computer input as follows:

$\Delta t = 1/4{,}000$

$W_1 = 5{,}000$, $W_2 = 700$, W_3 to $W_{11} = 530$ each, $W_{12} = 630$

$K_1 = 2{,}000{,}000$, K_2 to $K_{11} = 3{,}900{,}000$ each, $K'_{12} = 2{,}000{,}000$

$e_1 = 0.50$ $e_2 = 1.00$

Initial value of $V_1 = 12.4$

$Q = 0.1$

$J = 0.15$

F_2 can be equal to or greater than zero, but can never be less than zero.

If computer program #1 is being used the foregoing constitutes the required input data. If computer program #2 is being used, the following must also be specified:

$$K'_1 \text{ to } K'_{11} = 0$$

$$J' = 0$$

Step No. 9, Computation.—The values listed in step 6 are used as computer input for a computer that has been programmed in accordance with Program #1 or #2.

This has already been done for this particular problem. The permanent set s was computed to be 0.20311 in. per blow. (5 blows per in.) The computer was programmed to print maximum compressive and tensile forces in the pile. These were found to be as follows:

		Maximum Forces			
		Compression			Tension
At head of pile	(F_2) =	290,000 lb	=	18,600 psi	0
At middle of pile	(F_6) =	300,000 lb	=	19,250 psi	0
At point of pile	(F_{11}) =	405,000 lb	=	26,000 psi	0

Forces such as these are primarily of interest as a means of determining unit stresses during driving in the case of precast and prestressed concrete piles.

In the above example tension did not occur because $R_u = 100$ tons which is a moderately high value. Tension in piles occurs primarily during the early stages of driving when R_u is quite small.

The compressive stress at the point of the pile is higher than at the head because the fairly high value of R_u produces a reflected compressive wave which travels back up to the head of the pile. This upward wave overlaps the original downward traveling compressive wave and thus increases the stress at the point of the pile.

The calculated displacements D_1 to D_{12}, which apply to weights W_1 to W_{12}, as well as the plastic ground displacement D'_p are plotted in Fig. 12. Each dot on each curve represents a computed point. Fig. 12 gives a lot of information and is worthy of study.

For The Future.—Future investigators with more complete experimental data available may consider it worthwhile to include hysteresis (internal damping) in the pile as part of the calculation. This can be done by modifying Eq. 6 to read as follows

$$F_m = C_m K_m + B K_m \left(\frac{C_m - c_m}{12 \Delta t} \right) \quad \ldots\ldots\ldots\ldots (27)$$

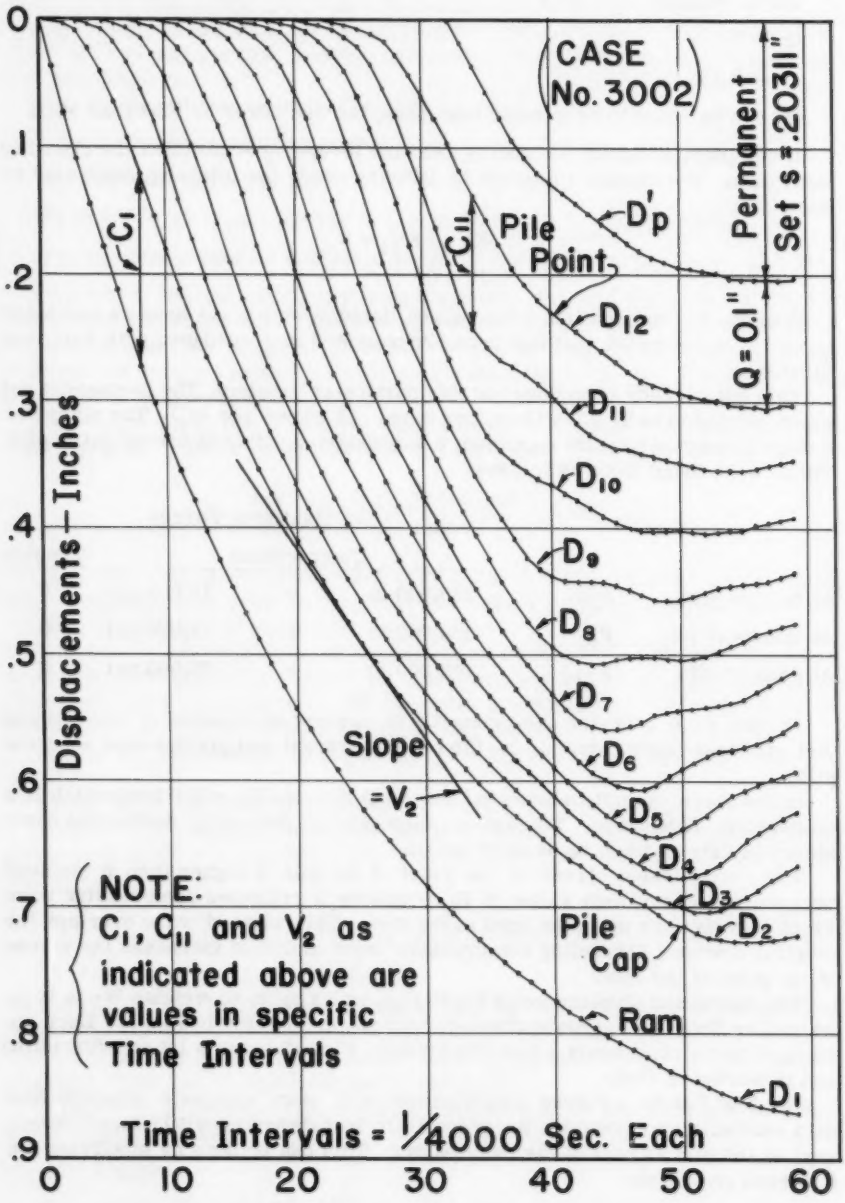

FIG. 12.—ILLUSTRATIVE PROBLEM

where B is a hysteresis or internal damping constant. A small value (such as 0.0002) should be assigned to B so as to produce a narrow hysteresis loop. Actually, the "loop" is a sort of spiral starting at the origin and returning to the origin when both C_m and $C_m - c_m$ become equal to zero.

The expression $\left(\frac{C_m - c_m}{12\,\Delta t}\right)$ in Eq. 24 is the instantaneous rate of spring compression. This corresponds in a general way to v_m and v_p in Eqs. 15, 16, 25, and 26.

A recommendation has been given previously to the effect that the calculation be stopped soon after maximum pile point penetration is reached as indicated in Fig. 8 and actually done in Fig. 12. Future investigators may want to continue beyond this point. If so, they should take note of the fact that Eqs. 15 and 16 give no damping at all when $(D_m - D'_m)$ becomes zero, even though the velocity may be considerable. To overcome this difficulty, Eqs. 15 and 16 can be used until $(D_m - D'_m)$ or $(D_p - D'_p)$ is first equal to Q, which corresponds to point A of Fig. 4. From this point on the following equations may be used instead of Eqs. 15 and 16, respectively:

$$R_m = \left(D_m - D'_m\right) K'_m + J' K'_m Q v_m \quad \ldots\ldots\ldots (28)$$

and

$$R_p = \left(D_p - D'_p\right) K'_p + J K'_p Q v_p \quad \ldots\ldots\ldots\ldots (29)$$

These two equations produce damping at all times except when v_m (or v_p) is equal to zero. Changing equations in this way is similar to the use of two different equations in routines #3 and #4.

It is also possible that Eq. 24 may in the future be found suitable for use with the capblock and cushion block, and thus replace routines #3 and #4. Before this can be done, suitable values for B in Eq. 24 would have to be determined experimentally.

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DISCUSSION

Note.—This paper is a part of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. SM 4, August, 1960.

STRUCTURE AND STRENGTH CHARACTERISTICS OF COMPACTED CLAYS[a]

Discussion by R. E. Olson and J. D. Scott

R. E. OLSON,[1] A. M. ASCE, and J. D. SCOTT,[2] M. ASCE.—The authors have made a very thorough and original study of the stress-strain properties of compacted clays. Their use of the soil-structural principles set forth by Lambe[3,4] seems to be a major contribution to our knowledge of the properties of unsaturated soils. Based on data now available, the writers agree that soil-structure has produced the changes in stress-strain properties postulated by the authors.

The authors used soil-structure concepts to try to explain the shape of the compaction curve, swelling pressures, and volumetric shrinkage. Since Lambe has described the formation of the soil structure in terms of double layer theory, we are forced to conclude that the engineering properties of compacted clays are determined by double layer phenomena.

The ultimate explanation of the engineering properties of all soils must be made in terms of the actual forces between the particles, and between the particles and the pore fluid. Lambe has used laboratory data and double layer theory to show that changes in the physico-chemical environment alter the engineering properties. He has relegated the negative pore water pressures to a position of dependence upon double layer phenomena. Actually, negative capillary pressures develop independent of double layer phenomena. These negative pore pressures are developed due to the interaction of water with the surfaces. This interaction is not a double layer phenomena, although, of course, it is a physico-chemical interaction. The writers, therefore, prefer to discuss the properties of unsaturated soils in terms of double layer phenomena and negative pore water pressures as two separate and independent variables.

It becomes of interest to determine which of the two variables has the major influence on engineering properties and which the minor. The authors have implied that the variation in dry density for compaction on the dry side of optimum is produced by changes in structure which in turn are induced by changes in the concentration of electrolyte dissolved in the pore water. They have used structure to explain other engineering properties. By inference, it appears that changes in electrolyte concentration have altered the engineering properties by very important magnitudes through their influence on soil structure.

To investigate the effect of dissolved electrolyte on the compaction curve, the writers performed three compaction tests on a clay soil at three different

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[a] October, 1959, by H. B. Seed and C. K. Chan.

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[3] "The Structure of Compacted Clay," by T. W. Lambe, Proceedings, ASCE, Vol. 84, No. SM 2, 1958.

[4] "The Engineering Behavior of Compacted Clay," by T. W. Lambe, Proceedings, ASCE, Vol. 84, No. SM 2, 1958.

pore water salt concentrations. The compaction tests were performed in a Harvard miniature mold using five layers, twenty-five blows per layer, and a 200 psi contact pressure. The soil was the minus No. 40 sieve portion of a Pennsylvanian shale from Fithian, Illinois, that is predominately illite. The index properties of the soil was determined.

INDEX PROPERTIES

liquid limit	40%
plastic limit	23%
plasticity index	17%
percentage finer than two microns	45%
activity (I_w/% finer than 2u)	0.38
specific gravity	2.76

Samples of the clay were made homoionic to calcium and were compacted at pore water salt concentrations on 1N, 1×10^{-2}N, and about 8×10^{-4}N. Concentrations were determined by direct chemical analysis. Samples were allowed to soak at least 24 hr prior to compaction. At a concentration of 1N, the density of the pore fluid is appreciably different from unity and it results in a fictitious increase in water content since water content is a weight ratio. To avoid this problem, a volumetric water content was utilized. The volumetric water content (w_v) is defined as the volume of the pore liquid (water plus dissolved salts) divided by the volume of solids. For salt concentrations of 0.01N and less, the water content may be obtained by dividing the volumetric water content by the specific gravity of the soil (2.76).

The results of the compaction tests are shown in Fig. 1. For compaction at a volumetric water content of 35% (w = 12.7%) a reduction in pore water salt concentration of 1,250 times increased the dry density by only 1 lb per cu ft. At optimum water content the range in densities is only 0.8 lb per cu ft with the 0.01N test having the highest density. On the wet side, the compaction curves appear identical. It is apparent that even very large changes in pore water salt concentrations are ineffective in altering the position of the compaction curve. No other physico-chemical variable was changed except the water content. The range in dry densities, from the lowest on the dry side, to the density at optimum, is apparently due to a change associated with the change in water content and not with variations in electrolyte concentration or any double layer variable.

Before discussing the alternate hypothesis, attention is called to the results of the unconsolidated-undrained triaxial tests that were performed using a cell pressure of 15 psi. The maximum deviator stress is plotted against compaction water content in Fig. 2. The concentration of salt in the pore water has had negligible effect here too. Stress-strain curves are shown in Figs. 3 and 4 for comparison with those obtained by the authors.

It appears that the shape of the compaction curve, and much of the rest of the authors' data, can be explained most effectively in terms of negative pore water pressures and the principle of effective stress.

The normal total stresses will be defined by the symbol σ, the normal effective stresses by the symbol σ and the pore water pressures by the symbol μ. The effective stress is given by

$$\overline{\sigma} = \sigma - \mu \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

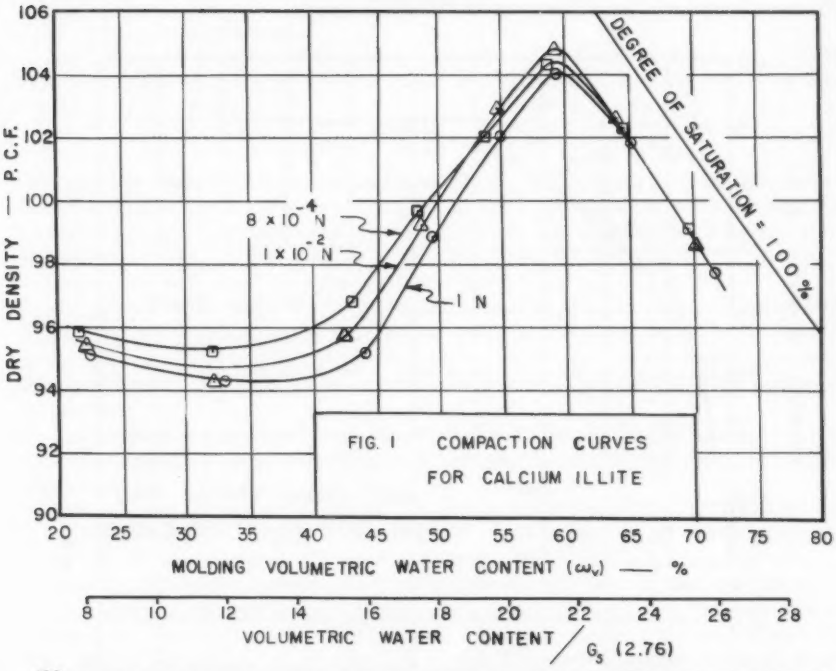

FIG. 1 COMPACTION CURVES FOR CALCIUM ILLITE

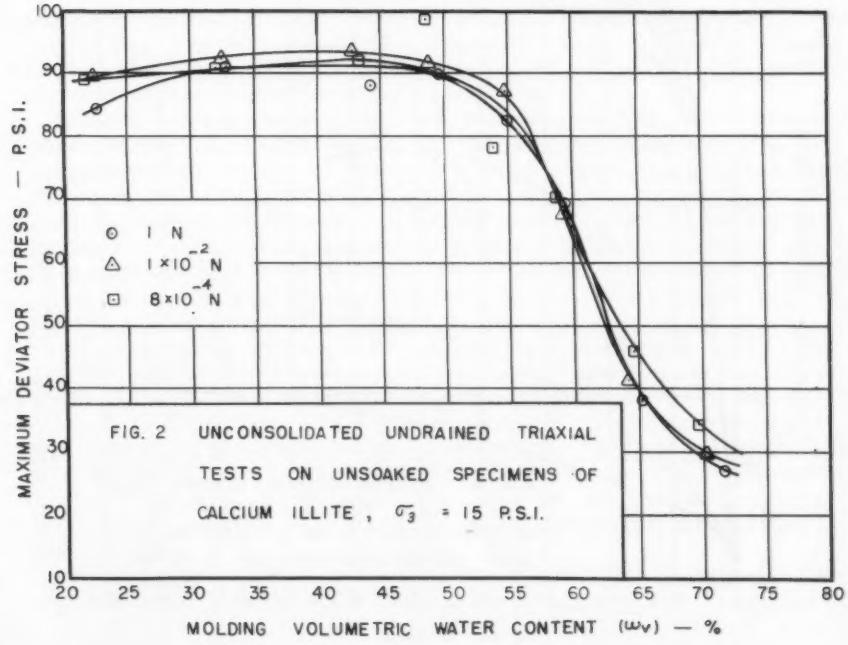

FIG. 2 UNCONSOLIDATED UNDRAINED TRIAXIAL TESTS ON UNSOAKED SPECIMENS OF CALCIUM ILLITE, σ_3 = 15 P.S.I.

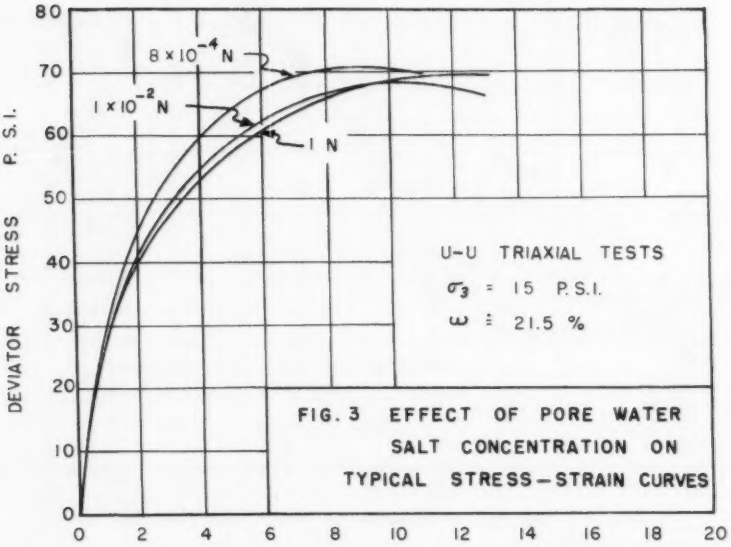

FIG. 3 EFFECT OF PORE WATER SALT CONCENTRATION ON TYPICAL STRESS-STRAIN CURVES

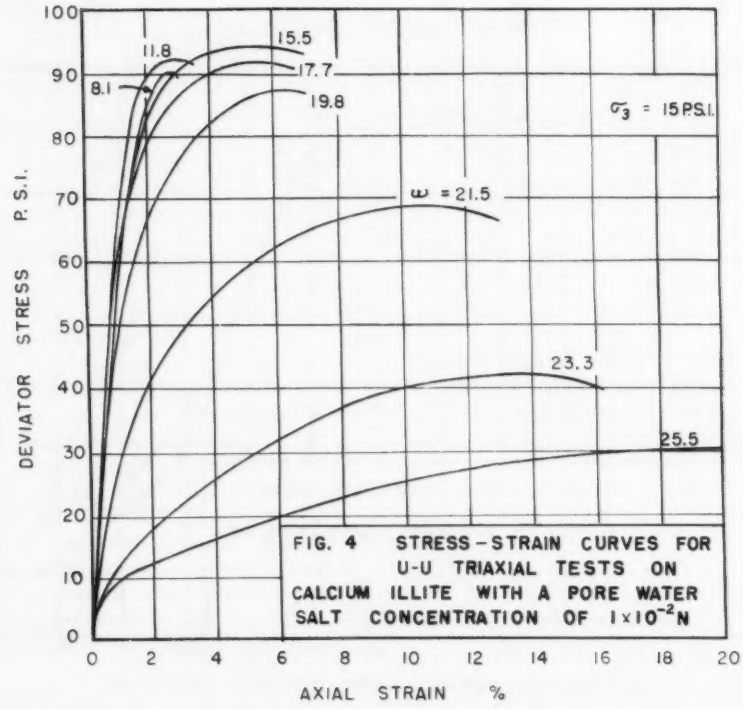

FIG. 4 STRESS-STRAIN CURVES FOR U-U TRIAXIAL TESTS ON CALCIUM ILLITE WITH A PORE WATER SALT CONCENTRATION OF 1×10^{-2} N

It can be shown that this equation applies to unsaturated soils so long as the water films are continuous.

It is also well known that there is a pressure differential across any curved interface between two substances. For a spherical interface the pressure differential is given by:

$$\Delta P = 2\, T_S/R \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which ΔP indicates the pressure differential across the interface. The pressure is highest on the concave side, T_S is the difference between the surface energies of the two substances in ergs per sq cm. (commonly the units are converted to dynes per cm and T_S is designated as the surface tension), and R refers to the radius of the spherical interface.

An unsaturated soil may be in equilibrium with pore water pressures that are either positive or negative. If the air voids are continuous with the atmosphere, then the pore water pressure must be negative. Conversely, if the pore water is continuous with an open water reservoir, then the pore air pressure is positive.

Let us first assume that the properties of a soil are effected by the effective stresses and the effective stress history. This assumption has been thoroughly supported by recent research, especially in shear strength.

For a given compactive effort, the density of a compacted soil will be related to the soil's resistance to compaction. The greater the compaction resistance, the lower the density. Any decrease in volume of a mass of soil involves interparticle movement. These movements demand that the interparticle shearing strength be overcome. Hence, any decrease in the volume of a soil mass, even under ambient stresses, involves shearing failure of the soil. The soil's resistance to compaction is essentially a shearing resistance and should be expressible in terms of such shearing parameters as ϕ and c. As yet (1960), it has not been possible to define the shearing properties of unsaturated soils in terms of these parameters because of the difficulties involved in measuring the negative pore water pressures.

The data presented by the authors can be used to estimate the negative pore water pressure. If a compacted soil is extruded from the mold, the total pressure becomes zero but the effective stress must have a finite value to maintain the given void ratio. Therefore, according to Eq. 1, the pore-water pressure must be negative. If the sample is soaked at constant volume the pore water pressure goes to zero and the total stress applied to maintain constant volume is numerically equal to the original negative pore-water pressure. This method is not exact since extrusion produces shearing strains and slightly alters the original negative pore-water pressures.

Fig. 5 indicates a plotting of the authors' Fig. 5. Applying the above principle, the negative pore pressure in the compacted specimens varied from almost zero on the very wet side to about 43 psi for compaction 3% dry of optimum. If the dotted line in Fig. 5 is used in place of the authors' line, this negative pore water pressure would be around 48 psi. Further compaction at 5%, or more, dry of optimum would result in negative pore-water pressures in excess of 70 psi and would give the soil a very appreciable shearing strength. Apparently, the resistance to compaction decreases markedly as the water content increases.

For compaction at water contents significantly less than optimum, the application of shearing stresses will result in the expulsion of air. The pore water

pressures will not change very much, even for dynamic compaction, because of the high compressibility of the air. As the water content increases, however, the shearing stresses will result in positive pore-water pressures during the time of action of the blow and the resistance of the soil to compaction will decrease further. Near the optimum water content, the air voids no longer are continuous with the atmosphere as shown by air permeability tests. At this point, the remaining air bubbles have no avenue of escape and the percentage saturation approaches a constant value as more and more water is added. Hence, for compaction on the wet side of optimum, the shearing resistance of the soil has little to do with the density. Much of the compactive force goes into excess pore water pressures. For wet side compaction with highly plastic soils, the same densities will be obtained, whether modified Proctor energy is used or whether the soil is pressed into the mold by hand.

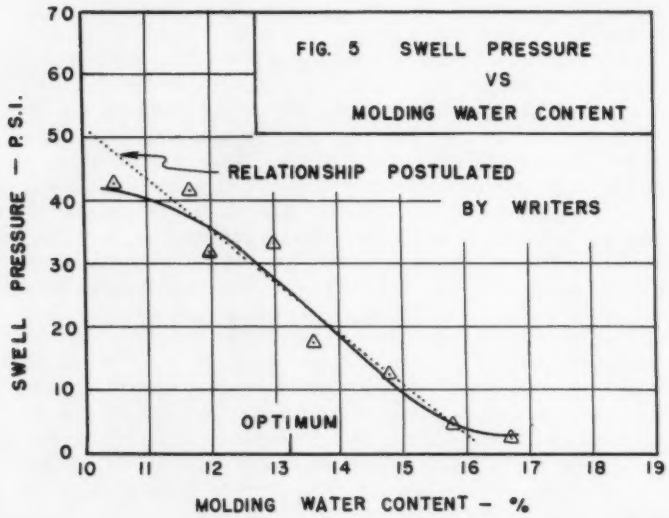

Fig. 1 shows that pore water salt concentration has essentially no influence on this part of the curve, as would be expected. It appears, therefore, that the shape of the compaction curve is more related to pore water pressures than to double layer phenomena though both play a part. Flocculation should have no effect on the shape or position of the compaction curve because the effective stress versus strain properties of a given soil appear to be independent of the soil structure (authors' Figs. 8 and 9).

The structure of a compacted clay depends upon the strength of the bonds that originally caused flocculation and upon the pore water pressures. As already pointed out by the authors, these bonds are sufficiently strong in some soils to maintain flocculation at all water contents. With other soils the flocculating force is so weak that a dispersed structure would result even for dry side compaction. For the large class of soils that fall in between these two extremes the compacted soil structure will be determined by the flocculation forces and by the negative pore-water pressures. The negative pore water pressures are important for the following reason. High negative pore water pressures result in high shearing resistance, low foot penetration, and hence

stability of the flocculated structure. The pore water pressures, during shear of compacted clays at constant volume, are related to steric interference between particles rather than to flocculating forces.

The drying shrinkage curve can be explained as follows: For compaction far on the dry side of optimum, the volume of the soil is maintained by a very high negative pore pressure. Drying the soil will result in a numerical increase in the negative pore water pressure and a very slight decrease in volume (the dry soil is highly incompressible). However, when the water film becomes discontinuous the soil undergoes an elastic rebound (rebound of bent plate shaped particles) and the net effect will be an increase in volume. The low compressibility of the above soil is due to steric interference between particles. If the water-content is increased and another sample is compacted, the soil will be in equilibrium at a higher density and with a negative pore water pressure that is closer to zero. Upon drying, the increase in effective stress will be much greater because the pore-water pressure started at a higher absolute value and decreased during drying to nearly the same value before the film snaps. This will result in greater decrease in volume during drying and consequently will eventually result in a net decrease in volume even after elastic volume rebound. The compressibility will also increase as the water content increases because, for a constant density, flocculated particles will cause more mutual interference than dispersed particles. The original negative pore-water pressure, therefore, has a dual effect. It changes the effective stress increase during drying and it alters the ability of the soil to resist changes in stress.

Soaking at constant volume prior to drying should not effect the shrinkage curves so long as no change in effective stress occurs during the soaking process. If the samples are soaked in the compaction mold and the pressure removed prior to drying, the stresses induced in the soil by the confinement of the mold will be partially removed, resulting in shearing deformations. Shearing deformations cause partial collapse of the flocculent structure and would increase the amount of shrinkage. The authors' data indicate such a change. This change is therefore tentatively attributed to partial remolding of the soil. The same type of argument can be used to explain the shape of the swelling curves under constant pressure.

In summary, the work of the authors in describing the shearing properties of compacted soils in terms of soil structure seems generally well founded. It appears that the principle of effective stress and the negative pore-water pressures can be used to describe the experimental data. The influence of double layer phenomena is considered to be of secondary importance.

A REVIEW AND EVALUATION OF SOIL-CEMENT PAVEMENTS[a]

Discussions by Ernest Zube, Earl J. Felt

ERNEST ZUBE,[1] M. ASCE.—The authors are to be complimented on the presentation of a very fine comprehensive paper which reviews the many viewpoints and practices concerning soil-cement. A summary of this sort serves as thought stimulus on a highly diversified subject.

It appears, however, that some elaboration is needed to clarify California's cement treated base (CTB) practice in relation to the design of bituminous pavement structural sections.

The highest classes of cement treatment (classes A and B) are intentionally designed to provide limited slab stiffness or rigidity to pavement sections. In a sense, CTB's might be analogized with a low grade concrete, since processed aggregate, fairly uniformly graded from coarse to fine, is employed. Controls are provided in the form of 7-day compressive strength requirements and grading limits on the aggregate. In addition, methods of mixing, placing, and compacting are clearly defined in the California specifications. All of this is calculated to provide a hardened layer under the pavement which will effectively minimize deflection under a wheel load and restrain plastic flow or deformation of the underlying soils.

Of the three variables (Traffic, R-Value and C-Value) embodied in our current design method for layer thickness determination, only the "C-Value," or Cohesiometer value as it is called, relates to the beam strength of the surfacing and base. This can be demonstrated by considering a load which is heavy enough to deform the surface, as shown in Fig. 1. In the lower portions beneath the load, it is considered that the principle deterrent to plastic flow of the soil in a lateral direction is interparticle friction, which is evaluated in terms of R-Value. However, if there is sufficient lateral deformation, in spite of the shear strength of the soil, then, by displacement, a wave will form in the surface outside of but adjacent to the load. From this, it can be visualized that the pavement and base are subject to tensile stresses and that the higher the tensile strength displayed by these layers, the greater will be the resistance to deformation. In addition, the stiffness of the surface materials immediately under the load tends to reduce stresses which promote plastic flow in the underlying soils. Since cohesion serves as a measure of a material's ability to resist tensile stresses, it is logical that this function be employed as a variable in design evaluation.

The Cohesiometer test, which forms the basis for the values used in the California design equation, is a relatively simple test. Referring to Fig. 2, the test consists of confining a specimen with two clamps which are connected at the bottom legs only by a hinge. The hinge coincides with the 4 in. diam of the

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[a] December, 1959, by J. K. Mitchell and D. R. Freitag.

[1] Superv. Materials and Research Engr., Calif. Div. of Highways, Sacramento, Calif.

specimen which is ideally 3 in. high. One of the clamps is held in a fixed position while the other has a beam attached to which the test loads are applied. By loading the end of the beam at a constant rate, the specimen eventually fractures at the hinge in a vertical direction. Since the hinged arrangement has the effect of placing the neutral axis at the bottom of the specimen, the entire central vertical plane is subjected to tension with increasing magnitude in the upward direction. The Cohesiometer value is expressed as the breaking load in g per in. of diam for 3 in. of height.

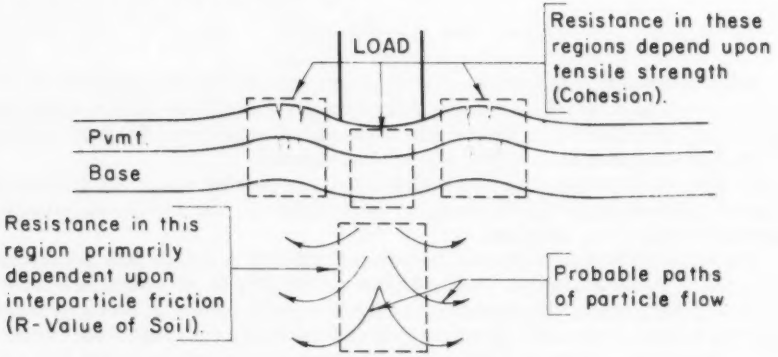
LOAD
Resistance in these regions depend upon tensile strength (Cohesion).
Pvmt.
Base
Resistance in this region primarily dependent upon interparticle friction (R-Value of Soil).
Probable paths of particle flow

FIG. 1.—REPRESENTATION OF PLASTIC FLOW PHENOMENA

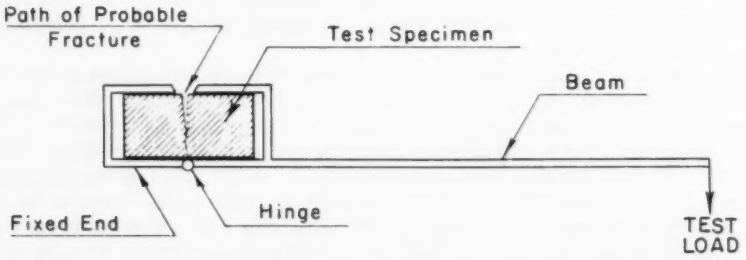

FIG. 2.—ARRANGEMENT OF THE APPARATUS USED IN THE COHESIOMETER TEST

For design purposes, Cohesiometer values have been established for each of the various materials employed in highway construction. These values have been determined from one or more of the following sources:

(1) Performance of a large number of Cohesiometer tests.
(2) Correlation of test track data.
(3) Correlation with experiences on California highways.

Table 1 lists the Cohesiometer values assigned to the most common materials.

Through the California design relationships and the values listed in Table 1, it is possible to determine the unit equivalency of cement treated base in terms of gravel (or soils). In other words, how much gravel is equivalent to 1 in. of CTB? The original test track studies established the relationship, for the

present design equation, that thickness of cover is proportional to $5\frac{1}{\sqrt{C}}$. By using this expression, the equation for unit gravel equivalent can be derived as follows:

Let T_g represent the thickness of gravel; C_g is the Cohesiometer value of gravel; Tx denotes thickness of other material; and Cx is the Cohesiometer value of other material. Then

$$\frac{T_g}{Tx} = \frac{1/5\sqrt{C_g}}{1/5\sqrt{Cx}} = \frac{5\sqrt{Cx}}{5\sqrt{C_g}} = 5\sqrt{\frac{Cx}{C_g}} \quad \ldots\ldots\ldots (1)$$

If Tx = 1 in. and C_g = 100 (C-Value for untreated soils or gravel). Then

$$T_g = 5\sqrt{\frac{Cx}{100}} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

Application of this equation to Class A CTB, which has a Cohesiometer value of 1500, gives: $T_g = 5\sqrt{\frac{1500}{100}} = 1.72$ in. of gravel per in. of CTB. Similarly, one in. Class B CTB equals 1.50 in. per in. and for comparison asphaltic concrete is 1.32 in. per in.

In this way, it is demonstrated how the California design method affects a reduction in thickness from granular bases when cement treatment is used.

TABLE 1.—COHESIOMETER VALUES.

Material (1)	Cohesiometer Value (2)
Asphaltic Concrete	400
Cement Treated Base, Class A	1500
Cement Treated Base, Class B	750
Road Mixed Surfacing	150
Soils, Aggregated Bases and Class C CTB	100

This is largely possible because, as previously mentioned, the use of processed aggregates and rigid quality control specifications provide reasonable assurance that a consistent product will be obtained. Due to the relatively heterogenous character of natural soils, it is problematical whether the California design approach of CTB could be applied universally to general soil-cement practice.

EARL J. FELT,[2] F. ASCE.—This discussion concerns, first, the data in Table 1, and second, the authors' evaluation of the structural performance of soil-cement.

The compressive-strength data in Table 1, referred to as "typical" for soil-cement mixtures containing 10% cement by weight, are very low when compared with the compressive strength data given in Table 2, and with those obtained elsewhere (47).[3] It would be a valuable addition to the data if the authors, in

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[3] Numerals in parenthesis, thus (1), refer to corresponding items presented in the Bibliography of the author's paper, and that of this discussion.

their closure, would present information on the density and moisture content of the specimens when molded, the method of curing, and the age and moisture content of the specimens when broken. All these factors greatly influence compressive strengths, and knowledge of them might help account for the relatively low strength values.

In studying the structural performance of soil-cement, the authors reviewed a considerable amount of field-test data. Their tabular summarization of the data is a useful addition to the growing volume of literature on soil-cement as a construction material for roads and airports. Unfortunately, however, they have gone beyond the bounds of reasonable speculation in certain portions of their paper. This remark refers to several assumptions in the paper and specifically to the columns headed "Flexible Pavement Thickness Required for Equal Performance" in Table 3, and "Flexible Pavement Thickness Required for Given Traffic" in Table 4.

The evaluation of the structural performance of pavements of different types is a very difficult task unless the pavements are built side by side on the same subgrade soil and carry the same traffic. The task becomes more difficult and the comparison less reliable when the actual performance of a pavement (soil-cement, in this case) is compared with the theoretical performance of a hypothetical pavement (flexible, in this case). This is the comparison the authors have made. They conclude that:

> "The subgrade protection provided by soil-cement pavements as currently designed and constructed is about the same as that expected from an equal thickness of conventional flexible pavement construction, for subgrades having California Bearing Ratio (CBR) values greater than 3."

The question is whether the data presented, as well as other data on the subject, support the conclusion. The paper implies that the conclusion pertains to both airfield and highway pavements, and that it applies to soil-cement pavements as "currently designed and constructed" and to "conventional flexible pavement construction." However, the authors consider practically no "currently constructed" soil-cement pavements, and their conclusion is based almost wholly upon the performance of test pavements and airport projects built in about 1942. Soil-cement, at that time, was a relatively new construction material, and construction practices were inferior to those which are now commonly accepted. On the other hand, the hypothetical flexible pavement being considered meets current requirements of the United States Corps of Engineers, Dept. of the Army, and is perhaps the highest type of granular pavement that can be built. It utilizes high-quality base materials compacted to 100% American Assn. of State Highway Officials (AASHO) T180 (modified) density. (AASHO Designation: T180-57, Standard Method of Test for The Moisture-Density Relations of Soils Using a 10-lb Rammer and an 18-in. Drop.)

Nevertheless, the comparison of performance of old soil-cement pavements with modern flexible pavements (even though the latter are hypothetical) would have some value provided the comparison were made on the basis of corresponding performance of the two. The violation of this essential requirement will be apparent to all who read the paper carefully, but may be overlooked by those who concentrate on the conclusions.

In column 13 of Table 3, headed "Coverages to Failure," 20 of the 50 entries include the sign $>$ (greater than), indicating that the number of test load applications listed had not produced failure. It appears that all corresponding entries in column 15, "Flexible Pavement Thickness for Equal Performance,"

should also include the sign >. Yet the authors have presented their comparison, in column 16, as though all soil-cement test sections had been tested to failure.

Analysis of Field Tests.—The data of Table 3, from Fort Belvoir (42), show a slight advantage for soil-cement. Actually, the advantage may be considerably greater than indicated in Table 3 since six of the eight soil-cement pavements did not fail. The quantitative comparison based on failure loading thus becomes meaningless. This is illustrated, for example, by the data in line 7 (CBR 4, wheel load 3000 lb, coverages to failure more than 201, actual soil-cement pavement thickness 8 in., theoretical flexible pavement thickness 5.5 in.). The flexible pavement thickness of 5.5 in. was selected from Corps of Engineer charts as being adequate to carry 201 coverages. However, the section probably would have carried easily more than 2700 coverages as did the 6-in. thick soil-cement section listed in line 5. Under such conditions a flexible pavement 8 in. thick would have been required, and the 2.5 in. advantage assumed by the authors in favor of the flexible pavement would no longer apply. If the soil-cement section supported 5,000 coverages, which it might have since the 6-in. section supported more than 2,700, the required flexible pavement thickness would be approximately 10 in. This same type of reasoning applies to many other entries in Table 3.

The data from Beltsville (13) and Fargo (13) appear to offer a better comparison, for loadings to failure are reported. These projects definitely favor soil-cement by several inches, and the CBR values of the subgrade are 11 to 9, respectively. Only in the test at Beltsville, where a construction joint was located near the traffic, was there any question about the superiority of the soil-cement. Here, a construction weakness coupled with a weaker subgrade as recorded in the Beltsville report resulted in early distress. Failure at Fargo occurred also at construction joint locations. Nevertheless, the soil-cement pavement has a thickness advantage of 4 in. or 5 in. over the hypothetical flexible pavement. These projects illustrate construction weaknesses appearing in jobs built in 1942 that are eliminated by the more precise construction methods now in use. Furthermore, Redus (40), of the Corps of Engineers, reported that no evidence of weakness at construction joints was noted at any of the airfields observed in his evaluation study conducted between 1952 and 1957.

In connection with the data from Barksdale (12) tests, it is significant that the in-place density of the soil-cement was only 92% of AASHO T134 standard (not modified) density. [(AASHO Designation: T134-57, Standard Method of Test for Moisture-Density Relations of Soil-Cement Mixtures. (Same compaction effort as AASHO Designation T99-57, Standard Method of Test for the Moisture-Density Relations of Soils using 5.5-lb Rammer and a 12-in. Drop.)] It is known that such a low density has a harmful effect on the quality of soil-cement (48), nevertheless the soil-cement base was about equal to the flexible pavement. Furthermore, the soil-cement did not consolidate or rut under the 20,000-lb wheel load traffic. In contrast, the Barksdale report notes that to prevent excessive consolidation of the flexible pavement, the upper base had to be compacted to 100% AASHO T180 (modified) density.

Next, consider the tests made in Great Britain. The Hurn (31) tests (CBR 5 to 6) favor soil-cement by 6 in. to 12 in. However, as pointed out by the authors, the CBR values for the subgrade soil involved appear too low, consequently, the theoretical thicknesses of the hypothetical flexible pavements may

be greater than they should be. The Dorset (27) tests (CBR 3 to 6) favor soil-cement by several inches for all tests in which a comparison is possible, that is, for all tests in which loading was continued to failure.

The only data in Table 3 that definitely appear to favor the hypothetical flexible pavement were obtained in the Somerset tests. However, as noted by the authors, the "computed flexible pavement thicknesses could be in considerable error" because of uncertainty regarding the CBR values.

In the second Somerset (33) tests, the authors base their theoretical flexible pavement thicknesses on CBR values of 14 to 17. However, the report shows that the CBR value (estimated from cone tests) decreased from a mean value of 18 for the surface soil at the test area to a minimum mean value of 9 at a 2-ft depth. In the absence of exact CBR data, the thickness comparisons in Table 3 lose their significance.

An important factor that should be considered in analyzing all the data presented in Table 3 is that only the Barksdale and Beltsville soil-cement test sections had bituminous wearing surfaces. More representative and improved performance of the pavements would have been obtained if they had been surfaced. The anticipated performance of the hypothetical flexible pavements is based on the assumption that they include a high quality bituminous wearing surface. Such a surface is absolutely necessary on a flexible pavement, for as noted in the Barksdale report, gravel bases (CBR 60 to 80) treated with calcium or sodium chloride rutted at less than 25 load repetitions.

Analysis of Airfields.—The data presented in Table 4 cannot be analyzed effectively because the flexible pavement thicknesses are based almost entirely on the traffic coverages up to the year 1944, and at that time few of the soil-cement pavements had shown distress. In 1953, the Portland Cement Association (PCA) made a survey of ten of these airfields. Airplane traffic had continued using the fields and many coverages had been added since 1944, thus, the comparable flexible pavement thicknesses given in Table 4, based upon 1944 traffic, are not particularly significant. As an illustration, consider Cochran Field, Macon, Georgia, where, in Table 4, traffic is listed as 10,000 coverages of 2-kip wheel loads. On this basis, a hypothetical flexible pavement of 4 in. is considered adequate, compared to the actual soil-cement pavement 7.5 in. thick. However, in 1953, DC-3's and DC-4's, with a maximum dual wheel load of 34,500 lb, were using the field. It is apparent that a 4-in. flexible pavement would no longer be adequate.

It may also be of interest, in connection with Table 4, to discuss Biggs Airfield where the average CBR is 26, but the range in values is from 5 to 47. Under the low range of subgrade support, the 8-in. soil-cement pavement carried thousands of 30-kip wheel loads. However, for a CBR of 5, a flexible pavement thickness of 25 in. would be required. In Table 4, a thickness of 6 in., which is for a CBR of 26, is considered adequate for the field. Blackland Field is another interesting study. Here a clay-gravel pavement base and subbase, apparently with a CBR from 24 to 100, failed under traffic. When 6 in. to 8 in. of the base was treated with cement the pavement gave good service. It is interesting to note that the authors find that a 3-in. flexible pavement would do as well.

Analysis of Highway Pavements.—Apparently, in arriving at the conclusion that equal thicknesses of soil-cement bases and of flexible bases are required if subgrade CBR values are more than 3, the authors considered very seriously the comments made in Highway Research Board Current Road Problems No. 8 (18). The conclusions in that Bulletin were not based on test data, but on the

best advice available when the original Highway Research Board Wartime Road Problems Bulletin No. 8 was written in 1943.

Actually, few flexible pavement highways are built to the unusually high standard specified by the Corps of Engineers for airport runways. For example, Highway Research Board Bulletin No. 58 (49) indicates that most State Highway Departments having a density specification require that subbases and bases be compacted to a density of between 95% and 100% of "standard" maximum density as determined by AASHO T99. (AASHO Designation: T99-57, Standard Method of Test for the Moisture-Density Relations of Soils Using a 5.5-lb Rammer and a 12-in. Drop.) Only two states indicate compaction control based on AASHO T180 as used by the Corps of Engineers, and they specify 95% of maximum. One might conclude that flexible highway pavements are not of as high quality as Corps of Engineers' airfield pavements. Thus, since soil-cement pavements, some of inferior quality, compared favorably with hypothetical airfield flexible pavements, it is reasonable to expect soil-cement to compare even more favorably with flexible highway pavements.

TABLE 1.—THICKNESS OF BASE AND SURFACE TO PREVENT DISPLACEMENT IN THE SUBGRADE SOIL.

R-Value of Subgrade	Gravel	Cement-Treated Base
	EWL 30,000,000, T.I.9	
15	26 in.	15 in.
30	21	12
45	16	9
60	10	6

Perhaps the best direct comparison of the performance of soil-cement pavements and high quality flexible highway pavements was reported by Hveem and Carmany (50) of the California Division of Highways. They show in their Fig. 30 that the thickness of cement-treated base (soil-cement) required to carry 10,000 repetitions of a 6,000-lb wheel load is 8 in. to 10 in., whereas the thickness of flexible pavement with a CBR of 100 is at least 13 in. Based upon these and other tests, Hveem, in 1948, developed a method whereby the thickness of soil-cement and the thickness of flexible pavement can be designed to carry any specific highway loading. In 1958, George Sherman (51) presented a revision of the 1948 method in which he shows that 1 in. of Class A cement-treated base (650 psi at 7 days) is equivalent to 1.72 in. of gravel. This is illustrated for various subgrade strengths in Table 1, taken from a nomograph in Sherman's paper.

As is to be expected, this table shows that the difference in thickness between a flexible pavement and a soil-cement pavement decreases as the quality of the subgrade increases but that the ratio between thicknesses is constant. Nevertheless, even though the R-value of the subgrade is as high as 60, a 4-in. increment still exists in favor of soil-cement in the California design. Additional data in the literature making a direct comparison between the performance of soil-cement and flexible pavements are limited. Whitehurst (52) reports a few deflections showing that under a 7,300-lb wheel load, the deflection of a soil-cement pavement was 45% (0.018 versus 0.041 in.) of that of a companion flexible pavement (probably not built to Corps of Engineer standards).

Hicks (53) presents data showing that the addition of cement to the top 6 in. of the aggregate base course of a flexible pavement reduced deflections from unsafe to safe values based on a selected allowable deflection of approximately 0.03 in. under a 7,500-lb wheel load. Mr. Hicks points out that the average CBR on this project was about 8, but that the CBR test does not detect resilience.

Hveem (54) presents considerable data on deflections of flexible pavements and of pavements with cement-treated bases. Although no direct comparison can be made of the performance of the two types of pavements, the data generally show that under a 7,500-lb wheel load the deflections of cement-treated base pavements were less than half of those for high-type flexible pavements even though the cement-treated base pavements were generally thinner. In a conclusion Hveem states, in regard to cement-treated base pavements, that "pavement structures of high slab strength need not be as great in overall thickness as sections utilizing lower slab strengths in order to perform satisfactorily in carrying traffic over resilient soils."

Flexible pavements, when built on high-strength subgrades, may be safe from extreme failure but it is not uncommon for them to consolidate (rut) under repeated medium-to-heavy traffic loadings. For example, rutting of 0.25 in. to 0.5 in. was recorded in the best flexible sections in the WASHO (55) test after 238,000 load repetitions, and in Kentucky (56) rutting up to about 0.5 in. has been reported. To the writer's knowledge, no rutting of soil-cement highway pavements has ever been reported in the literature.

Laboratory information on densification of flexible bases and of soil-cement bases bearing out the above observations is provided by Colley and Nowlen (57). They show that a 6-in. thick granular base compacted to 100% of "standard" AASHO T99 density may be reduced in thickness by 0.2 in. or more under 450,000 load repetitions whereas practically no reduction in thickness occurs in a soil-cement base.

Concluding Remarks.—An analysis of the unclouded portion of the data in Table 3 indicates that the actual performance of soil-cement airfield pavements was superior to that of the theoretical performance of high-type flexible pavements of equal thickness. This was true at Fort Belvoir; Hurn and Dorset Fields, Great Britain; Beltsville; and Fargo, on all types of subgrades having CBR values from about 2 to 11 even though the quality of the soil-cement pavements was frequently inferior to the quality of present-day soil-cement pavements. The flexible pavement thicknesses listed in Table 4, for comparison with the soil-cement pavement thicknesses are based largely on traffic coverages up to the year 1944. However, the soil-cement airfields continued to carry traffic for years after 1944, and a number of them are still in operation carrying loads far heavier than originally intended. Accordingly, the comparisons given in Table 4 have limited value.

The superiority of soil-cement pavements over flexible pavements is a reasonable finding since soil-cement has much greater compressive strength (15), beam strength (15), and shear strength (58) than untreated granular materials. Consequently soil-cement pavements have greater load-carrying capacity and greater resistance to consolidation and rutting under high tire pressures.

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COMMENTARY ON PLASTIC DESIGN IN STEEL: CONNECTIONS[a]

Discussion by Hashim H. Hamzawi

HASHIM H. HAMZAWI.[1]—While discussing tapered haunches, it is stated that "It has been shown that if the plastic moment of resistance is adequate at the rolled section (section 1 Fig. 8.6) and at the re-entrant corner (section 2 Fig. 8.6), there will be no necessity of checking at other sections along the haunch." This statement, which was made when only bending of the haunch was considered, is questioned by this writer.

For the haunch shown in Fig. 1, between sections 1 and 2, the same assumption is made that the original paper makes, namely, that the cross section is symmetrical and that the centroid is at mid depth. The haunch then can be treated as a cantilever beam (Fig. 2) fixed at A (relative to B) and acted upon by a concentrated load Q and a moment M at the end B. The moment M can be clockwise or counterclockwise and assumed positive in the direction shown in the figure (clockwise).

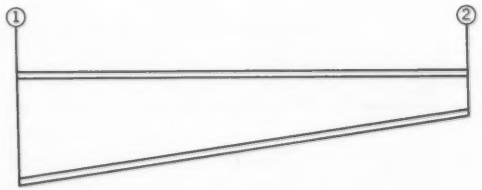

FIG. 1

The resisting plastic moment M_{px} of the member at any cross section can be expressed as

$$M_{px} = \frac{\sigma_{yp}\, w}{4}\left(d_{wA} - 2\,n\,x\right)^2 + \sigma_{yp}\, b\, t\left(d_A - t - 2\,n\,x\right) \ldots\ldots (1)$$

where σ_{yp} is the yield point stress. The external moment M_{qx} at any cross section x is

$$M_{qx} = Q(L - x) + M \ldots\ldots\ldots\ldots\ldots (2)$$

Equating Eqs. 1 and 2 and solving for x (the point of tangency between the two curves of Fig. 2), which will be equal to p (the distance of the plastic hinge

[a] April, 1960, Progress Report No. 6 of the Joint WRC-ASCE Committee on Plasticity Related to Design.

[1] Asst. Dean, College of Engrg., Univ. of Baghdad, Iraq.

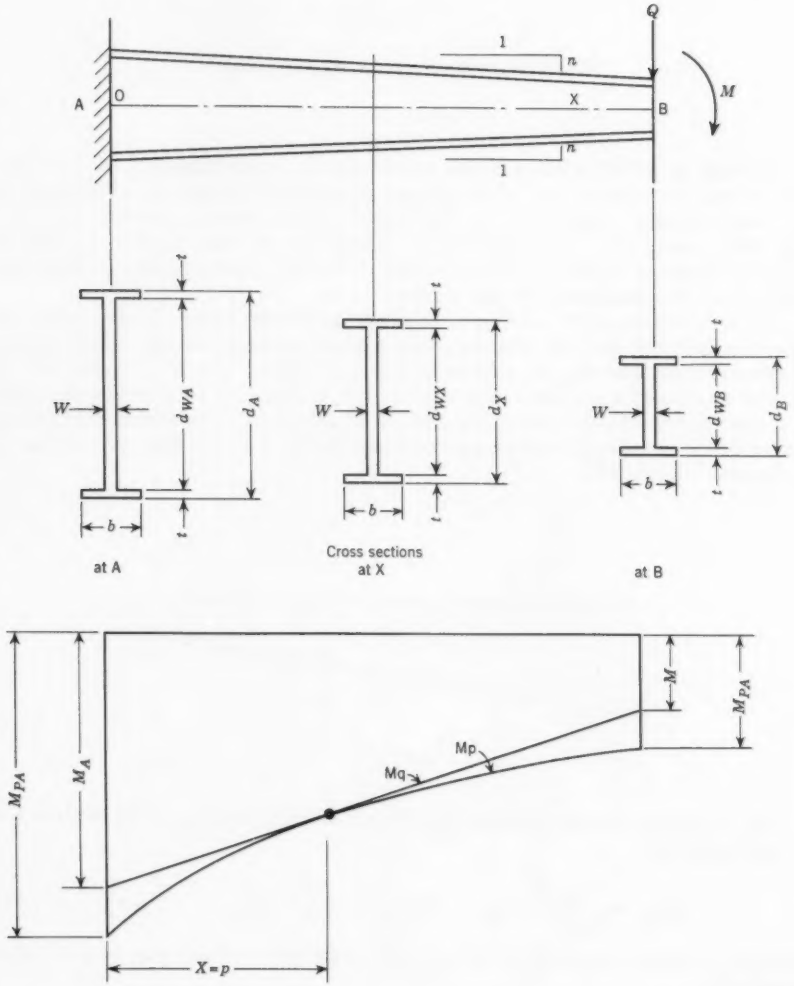
Q
1
n
A
O
X
B
M
n
1
t
W
d_{WA}
d_A
b
t
at A
t
W
d_{WX}
d_X
b
t
Cross sections at X
t
W
d_{WB}
d_B
b
t
at B
M_{PA}
M_A
M
M_{PA}
Mp
Mq
X=p

FIG. 2

from the end A) it is found that

$$x = p = \frac{\rho n d_{wA} + 2 n t - \frac{Q}{\sigma_{yp} b}}{2 \rho n^2} \quad \ldots\ldots\ldots\ldots (3)$$

and

$$Q = \sigma_{yp} b n \rho d_{wA} + 2t - 2 \rho n L + \sqrt{\rho^2 d^2_{wB} + 4 t \rho d_{wB} + 4 t^2 \rho - \frac{4M}{\sigma_{yp} b}} \ldots (4)$$

in which $\rho = \frac{w}{b}$ substituting $\frac{1 - k}{k} \frac{d_B}{2 L}$ for n where $k = \frac{d_B}{d_A}$, $d_B - 2t$ for d_{wB}, β_B for $\frac{t}{d_B}$ and the value of Q given by Eq. 4 in Eq. 3 gives

$$p = L \left\{ 1 - \frac{k}{1 - k} \sqrt{\left[4 \beta_B \left(1 - \beta_B \right) \left(\frac{1}{\rho} - 1 \right) + 1 \right]} - \frac{4 M}{\sigma_{yp} w d_B^2} \right\} \ldots (5a)$$

or

$$p = \frac{1 - \left[1 + \left(C_B - 1 \right) \quad 1 - \sqrt{\frac{M}{M_{pB}}} \right] k}{1 - k} \; L \ldots\ldots\ldots (5b)$$

where

$$C_B = 4 \beta_B \quad 1 - \beta_B \left(\frac{1}{\rho} - 1 \right) + 1 \quad \ldots\ldots\ldots\ldots (6)$$

Examining Eq. 5(b) it is found that for values of

$$O \leq k \leq \frac{1}{1 + \left(C_B - 1 \right) \quad 1 - \sqrt{\frac{M}{M_{pB}}}} \quad \ldots\ldots\ldots\ldots (7)$$

the value of p lies between L and O indicating that the plastic hinge develops at an intermediate section between the ends A and B due to the applied moment M and the corresponding Q (Eq. 4). Therefore, it is evident that even though the member is safe for bending at the ends, it may fail at any other intermediate section where the plastic hinge develops.

PILE CONSTRUCTION IN PERMAFROST[a]

Discussion by Frederick E Crory

FREDERICK E. CRORY,[1] A. M. ASCE.—The author has presented an effective and informative review of the construction problems and foundation practices in the permafrost areas of Canada. Such contributions are currently in great demand in view of the increased emphasis in construction throughout the northern regions. Inasmuch as some of the information contained in the paper, particularly pile design and installation methods, may have wide application by the profession, it is felt that possible emphasis and discussion by others similarly engaged in cold-regions engineering is required.

The Arctic Construction and Frost Effects Laboratory (ACFEL) of the U. S. Army Engineer Division, New England, is engaged in an extensive program to determine the most economical and reliable foundation designs for various types of structures and foundation conditions in the Arctic and Subarctic. Included in this program are comprehensive studies to obtain design criteria for pile foundations in permafrost. These studies are carried out at a field station near Fairbanks, Alaska, and in conjunction with existing and proposed structures throughout Alaska. This discussion, therefore, reflects the experience and knowledge gained through ACFEL's field and laboratory investigations.

Under the heading, "Design Considerations," and as shown in Fig. 4, the author's implication that "when a pile is first placed, it transfers its load to permafrost by end bearing," may be misleading or erroneous. The apparent resistance to further pile penetration at the bottom of a steam thawed hole is not indicative of the actual sustained point bearing capacity available to support the design load. Frozen soils, especially those containing large volumes of ice, may be quite capable of resisting large dynamic forces, yet, under relatively low sustained stresses, may flow plastically, particularly at temperatures approaching 32°F. Inasmuch as the temperatures at the bottom of steam-thawed holes approach and, in some cases, may exceed 32° F for a brief period following pile installation, the writer feels that the interim point bearing capacity of piles installed by this method should be used with reservation. It is suggested that the point bearing capacity of piles, installed by steam or by any other installation method, be determined by field load tests if the point bearing capacity must be utilized to maintain construction schedules.

The author states that "the design problem is two-fold—first to predict the thawing effect that the structure will have on permafrost during the life of the building, and second to determine what tangential adfreezing strength can be safely assumed." The depth of thaw beneath and adjacent to a ventilated pile foundation is not only dependent on the loss of heat from the structure, but also

[a] December, 1959, by J. A. Pihlainen.

[1] Engr., Arctic Constr. and Frost Effects Lab., U.S. Army Engr. Div., New England, Waltham, Mass.

on the thaw resulting from disturbance to the surface cover by clearing or construction equipment, the pile installation method, and from drainage. All of these factors have been adequately covered by the author, but they have been included here to emphasize their importance in the design problem. Undoubtedly, there are other thaw factors peculiar to certain locations or foundation designs that should also be considered, however, the writer wishes to discuss only one additional thaw factor—that of solar heat. Though indirectly responsible for the thaw resulting from some of the previously mentioned factors, the thawing attributed to the heat absorbed from direct or reflected solar radiation has a distinct effect upon the overall thawing that will occur during the life of the structure.

The effect of solar heat upon three originally similar adjoining test sections at the Fairbanks field station, after changing the surface cover conditions of two of the sections, is shown in Fig. 1. Section B was hand cleared of all trees and brush, but retaining the moss and peat surface cover. Section C was mechanically stripped of all surface cover, exposing the natural underlying silt. Periodical clearance of new growth in Sections B and C was required to maintain the desired surface cover. The cleared and stripped areas originally (1946) had essentially identical surface cover consisting of trees, brush, moss and peat, as shown in Section A, the control section. The areas were equally exposed and did not contain structures or other facilities that might have introduced extraneous heat or shading. The resulting thaw, determined by probings after 5 yr and 10 yr, is shown beneath each section in Fig. 1.

Thawing of permafrost is normally greatest on southern exposures where direct solar radiation, coupled with reflected radiation from the building, produces maximum heat absorption during the prolonged summer daylight hours that are peculiar to northern latitudes. Conversely, on northern exposures the permafrost may actually aggrade, due to the shading effect afforded by the building. Improperly designed roads adjacent to the foundation, especially on southern exposures, may absorb tremendous quantities of direct or reflected radiation and initiate detrimental thawing action of the foundation permafrost. This differential thawing and subsequent surface subsidence, of northern and southern exposures, may completely change the desired surface drainage characteristics, thereby further aggravating the thawing action beneath the foundation.

The importance of preserving permafrost not only at a design depth, but at relatively shallow depths, can be easily depicted by plots of ground temperature isotherms with time. The annual variation of ground temperatures for an average fiscal year, from the ground surface to a depth of 35 ft in an undisturbed, treeless, moss or tundra-covered area of continuous permafrost, would approximate those shown in Fig. 2(a). After a hypothetical summertime pile foundation installation, in which considerable heat was introduced through surface cover disturbance and the pile installation method, the isotherms would then reflect the effect of construction similar to that shown in Fig. 2(b). Note the general overall warming that has occurred in the permafrost, and especially the increase in depth of the active layer as compared to the same preconstruction periods in Fig. 2(a). Because of the greater depth of thaw resulting from the construction, complete freezeback of the active layer might still be incomplete in January, and possibly would not occur until some time in late February or March as shown in Fig. 2(b). The delayed frost penetration due to the increased depth of thaw will reduce both the intensity and duration of the cold penetration into the permafrost. Thus, appreciably less heat will be extracted

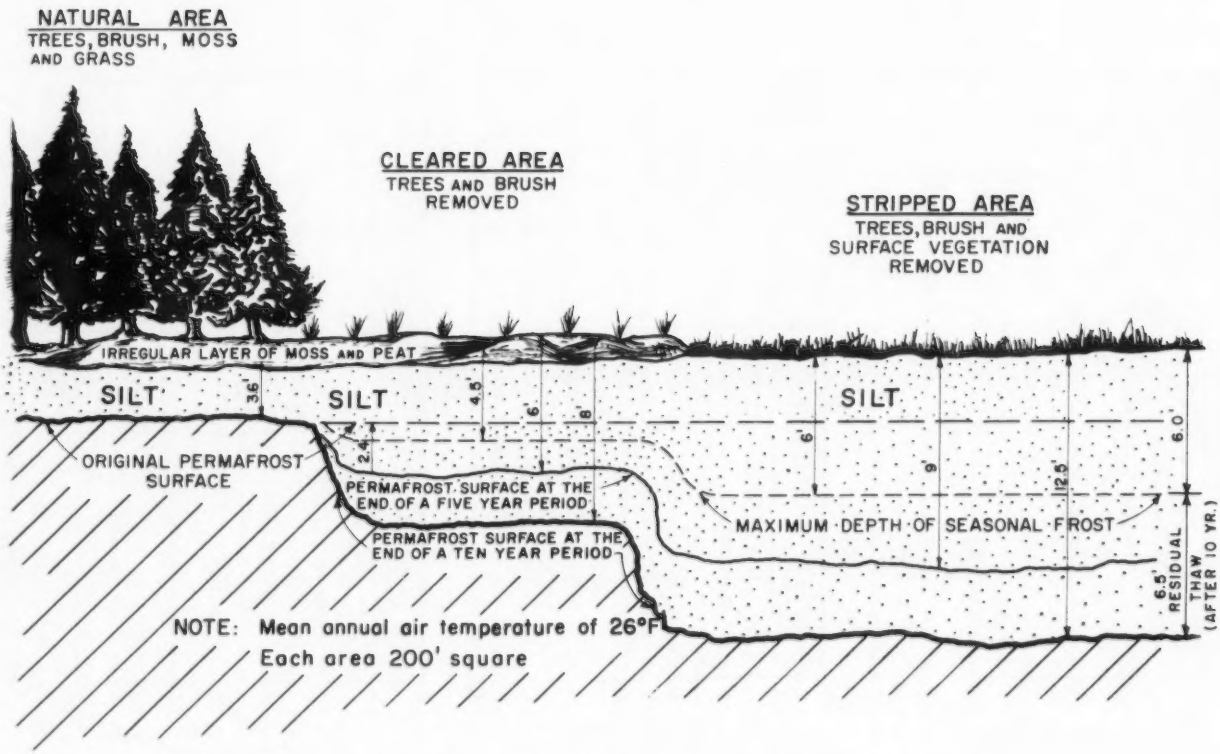

FIG. 1.—MEASURED DEGRADATION OF PERMAFROST IN FROST SUSCEPTIBLE SOILS BELOW DIFFERENT SURFACES IN A SUBARACTIC REGION

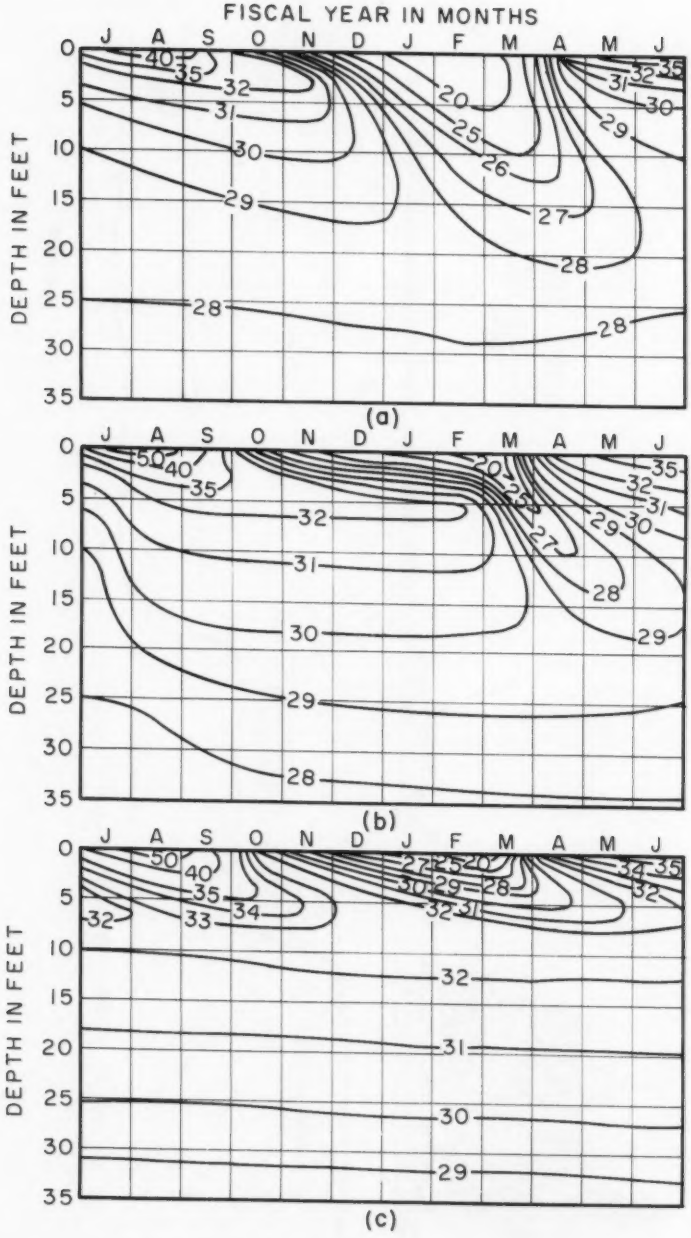

FIG. 2.—GROUND TEMPERATURES IN °F WITH DEPTH

from the underlying permafrost in contrast to that which was formerly readily accomplished to much greater depths before construction.

Assuming that additional thawing progresses over the next five yr until, possibly during a relatively mild winter with considerable snow accumulation or after a wet but unusually warm summer, or both, the active layer does not completely refreeze and a residual thaw zone results. The permafrost temperatures then become relatively stable and considerably warmer, as shown in Fig. 2(c). The permafrost temperatures over the following years will continue to slowly approach the thawing point, relying only upon the reserve of cold expendable from greater depths and the insulating effect of the thawed active layer to retard further thawing action. The residual thaw condition shown in Fig. 2(c) is identical to the previously described Sections B and C shown in Fig. 1.

The determination of the ground temperatures that will exist with time is of the utmost concern in pile design, inasmuch as the tangential adfreezing strength varies with temperature. If permafrost preservation is effective, wherein the active layer completely refreezes each winter, the design temperature distribution might be assumed similar to that shown during September or October in Fig. 2(b). If thawing is excessive, as in the case of the inadequate preservation condition shown in Fig. 2(c), the permafrost temperature gradient with depth will be quite small and near the thawing point for a considerable depth. The contrast in the permafrost temperatures available to effect an adequate adfreeze strength in Figs. 2(b) and 2(c), especially at the shallow depths in permafrost, clearly indicate the necessity for permafrost preservation.

In regard to the second part of the design problem, the determination of the design adfreeze strength, the author has referenced the early Russian investigations which are now known to be dangerously misleading. The adfreeze strengths reported in these references cannot be directly applied to pile design. The desired design pile length in permafrost cannot be obtained by simply dividing the design load by the circumferential pile area per inch and the adfreeze strength corresponding to the assumed design temperature shown in the author's Fig. 6. Even when utilizing corrections to the adfreeze strength because of the soil condition, or employing greater than normal factors of safety, the design criteria is inadequate. The shortcomings in the use of these data were evidenced in early pile tests performed by ACFEL, and have not been used for pile design for many years.

The error in the use of the author's referenced adfreeze strength data lies in the fact that the tests were conducted at rapid rates of loading, with little regard to the ensuing deflection or progressive creep deformation, which occurred at stresses far below the ultimate strengths reported. Tsytovich[2] reports that the Russians now recognize the effect of time and deformation in the determination of the sustaining stresses required for design application. This approach is reported to produce the required adfreeze strength values which, after proper corrections for variance in the soil condition, may be directly applied to foundation design. The author's Fig. 6, has been reproduced as the writer's Fig. 3 to clearly indicate the contrast in the magnitude of the adfreeze strengths, as reported in the earlier investigations, with those recently (1958) reported by Tsytovich. The earlier Russian investigations are still useful, however, as they represent a valuable source of relative information on the inter-relationship of the parameters influencing adfreeze strength.

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[2] "Bases and Foundations on Frozen Soil," by N.A. Tsytovich, Academy of Sciences, USSR, Moscow, 1958.

Although field and laboratory research has disclosed considerable information on the behavior characteristics of the predominant frozen soil types, considerably more research is still required to encompass more of the perennially frozen soil types and conditions. Even after exhaustive research, the design criteria should not be expected to be universally applicable to all permafrost foundation conditions. The writer feels that the design practice of piles in permafrost should be quite similar to that employed in the design of temperature zone friction piles in unconsolidated soils. The pile design assumptions are based on the information available from soil logs, laboratory strength tests, adjacent structure performance, etc., and then, after installation of the design pile, the bearing capacity should be verified by load settlement tests. Pile load tests in permafrost, though similar to temperature zone methods, differ in some important respects.

Pile load tests in permafrost should be conducted at that time of the year when the ground temperatures are at their warmest (normally in the fall of

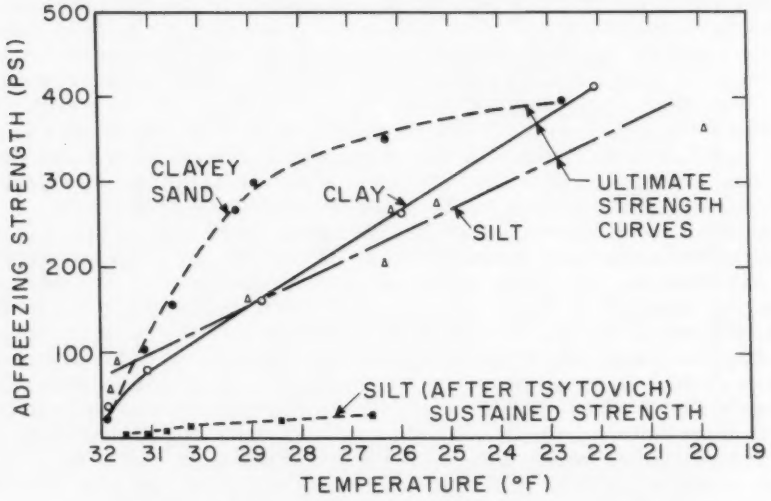

FIG. 3.—COMPARISON OF THE EFFECT OF TEMPERATURE ON ULTIMATE AND SUSTAINED ADFREEZING STRENGTHS

the year) if the active zone is capable of refreezing completely each winter. If a residual thaw zone exists at the site even before construction begins, testing may be accomplished at any time of the year, provided, of course, that the frozen active layer is eliminated from contributing to the pile capacity. The test pile should be loaded at relatively small increments with the interval between load application being one or more days. Slow rates of loading permit proper analysis of the test data to determine the load at which continuing settlement or creep commences. Creep is especially pronounced in most frozen soils as the temperature approaches thawing, often occurring at only a small fraction of the ultimate strength attainable in the test. Because a constant creep of only 0.01 in. per day would equal more than 3 ft of settlement in 10 yr, the accuracy required to observe settlement dictates the use of deflection gages accurate to 0.001 in. or better. These gages must, in turn, be supported by a stable instrumentation framework protected from exposure and frost heave or

subsidence. Arctic pile testing is often both costly and difficult, especially when trying to obtain a high degree of accuracy; however, testing is the only reliable method of predicting the success or failure of the pile design.

Regarding the placing of piles in permafrost, the author states that "Both methods," steam jetting and drilling, "should be reviewed before a final foundation is chosen since each method has special merit when considered to final costs and permafrost conditions." The writer feels that the same review should be given to all of the possible installation methods that might be advantageously employed. Many of these installation methods have been described by Kitze,[3] while still other methods, utilizing recent advances in soil exploration techniques in permafrost, should be considered. A more comprehensive list of the installation methods which might be reviewed is as follows:

1. Steam jetting—Steam thawing hole in permafrost and "driving" pile, normally wood, into thawed hole—described by author and Kitze.[3]
2. Rotary drilling—rotary drilling utilizing various bits, barrels, etc. with water or air, slurried back.[3,4]
3. Dry—Augering—holes augered dry and slurried back—described by the author and others.[3,5,6]
4. Driving—conventional or modified temperate zone pile driving techniques.[3,7]
5. Churn drilling and driving—churn drilling undersized hole, all or part way into permafrost—then drive pipe pile with churn drill.[3,7]
6. Various combinations of above—for example, coring, steaming or augering through colder "permafrost" in early spring, then driving (structural or pipe sections) to greater depths in warmer permafrost.

In contrast to the author's statement that piles are not driven into permafrost, structural or pipe sections may actually be driven by conventional equipment, under certain conditions, even in areas of continuous permafrost.[3,7] Recent advances in both techniques and equipment used in rotary drilling in permafrost, by wash boring or coring, offer an effective installation method not only in frozen fine grain soils and ice but in the frozen glacial tills containing cobbles and boulders.[3,4] Each installation method has special merits, not only in relation to the final costs and permafrost conditions, but also to the sustained adfreeze bond strength developed by each method.

Pile foundations with an air space offer a potentially excellent foundation in permafrost. The full potential, however, may never be realized unless proper regard is given to the many factors affecting the foundation material. The designer must prevail upon the experience and research of others to direct his judgment in view of the many uncertainties which still exist in the design criteria. Additional responsibilities must be placed on the foundation designer in permafrost areas to insure that the design conditions are preserved throughout the intended life of the structure. These additional responsibilities include the

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3 "Installation of Piles in Permafrost," by F. F. Kitze (presented at 8th Alaskan Science Conference, Anchorage, Alaska, September, 1957). ACFEL Misc. paper MP-18.

4 "Core Drilling in Frozen Ground," U. S. Army Engr. Waterways Experiment Sta. Tech. Report No. 3-534, Vicksburg, Miss., January, 1960.

5 "How Raber and Kief drilled 900 Holes in Permafrost in the Arctic in Midwinter," by Hank Pearson, Pacific Builder and Engr., November, 1956, pp. 58 and 59.

6 "Freezing of Slurry around Wood and Concrete Piles," by R. F. Scott, ACFEL Misc. Paper MP-13, May, 1956.

7 "Permafrost Design and Construction within the Alaska District Corps of Engineers," by E. L. Long (presented at ASCE Conf., Fairbanks, Alaska, October, 1957).

supervision and inspection necessary, not only during the site selection and soil exploration period, but during the construction and maintenance periods as well.

PERMAFROST ASPECTS OF HUDSON BAY RAILROAD[a]

Discussion by K. A. Linell

K. A. LINELL,[1] F. ASCE.—This paper illustrates graphically the improvements in civil engineering knowledge, facilities and techniques which have occurred over the past 50 years. In reading the paper the writer is greatly impressed by the courage and ingenuity required to initiate and carry on so difficult a project with the facilities which were available for this work in the early part of this century. These facilities were primative by present standards. Further, the knowledge concerning construction on permafrost which is summarized in the conclusions of the paper was not then available. The difficulties of making cuts and fills and of draining miles of muskeg by hand labor, and even of chopping out permafrost by hand in absence of any mechanical equipment—in the earlier days even lacking horses and mules—are difficult to visualize. Today, also, it is hard to imagine carrying out such a project without the aid of aerial photography and of photogrammetric techniques. It took great courage and determination to build this railroad in spite of opinions that it was not practical and in spite of the very slight knowledge of permafrost then available.

Of interest is the fact that the organic deposits encountered were not removed. Indeed, the embankments were built directly upon them and apparently much of the embankments themselves were made of organic material. This provides an interesting comparison with present-day practice in Scandinavia where peat is compressed in factories and laid in railroad subgrades for purpose of reducing frost penetration and heave. The peat acts to reduce frost penetration because of its high moisture holding capacity and consequent high latent heat content. However, in Scandinavia permafrost is not present under the railroads.

Regarding the effectiveness of explosives in frozen ground, the author notes that the efficiency factor of explosives in blasting frozen materials is comparatively low. It is of interest to note that during the last few years the Snow, Ice, and Permafrost Research Establishment of the U. S. Army Corps of Engineers has sponsored extensive studies to investigate and improve the efficiency of blasting in frozen materials. A paper by Clifton W. Livingston[2] summarizes the results of some of these studies.

Charles has cited 3 axioms: (1) avoid excavation in permafrost wherever practical, (2) avoid disturbing the natural surface insulation, and (3) endeavor to retain the permafrost condition. Experience of the U. S. Army Corps of Engineers in Alaska and Greenland has shown that cuts are a tremendous source

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[a] December, 1959, by J. L. Charles.

[1] Chf., Foundations and Matts. Branch, New England Div., Corps of Engrs., Waltham, Mass.

[2] "Fundamentals of Arctic Blasting," by Clifton W. Livingston, Proceedings, ASCE, Vol. 86, No. CO 1, February, 1960.

of difficulty, even in areas which are much colder than Churchill and in which the permafrost is, therefore, more easily maintained. The construction of roads and railroads in form of embankments on the natural ground is not always readily accepted by engineers who have been trained to balance cuts and fills, but in cold regions this has the advantage of providing improved drainage, of assisting in snow drift control and, in permafrost areas, of providing a protective cover over the permafrost, as has been brought out by the author. Disturbance of the natural surface insulation may result in very serious thawing and erosion, even in the very cold areas of the northern Alaska coastal zone. However, retention of a reasonably stable permafrost condition may be very difficult in borderline permafrost areas, particularly under pavements.

The same difficulty with heave of piles as described by the author has been encountered in Alaska. The U. S. Army Corps of Engineers Arctic Construction and Frost Effects Laboratory has been making an engineering study for some years to provide criteria and methods by which this problem may be avoided. Tentatively it appears that if a pile is once solidly frozen back and its depth of embedment in permafrost is 10 ft or more, the pile should not heave if the permafrost level is maintained. In order to insure rapid freeze-back, Corps of Engineer practice during recent years in Alaska has been to rely on augering rather than steam thawing because of the much smaller amount of heat introduced into the ground. Also in borderline permafrost areas it has been found desirable to install the piles during the spring period, approximately March, April and May, during which the greatest "reserve of cold" is present in the ground and when the most rapid and positive freeze-back can be obtained. It should be noted that the present-day specialized augering equipment was not available during the period of construction of the Hudson Bay Railroad, except for the most recent branch lines.

Very flat slopes were used in cuts in permafrost. In seasonal frost areas gravel blanketing of slopes which are unstable as a result of seepage or of frost melting conditions in the spring has been used with excellent success. It would be of interest to know whether this method has been attempted in any of the work on the Hudson Bay Railroad system.

With respect to the problem of soft soils pumping through the subbase materials, it is considered by many that sand offers a greater resistance to this type of contamination than gravels. It would be of interest to learn the author's observations on this point.

Plate No. 2 in the author's paper shows layers of ice 0.3 ft and 0.5 ft thick at 138 ft and 146 ft, respectively, below the surface at Churchill. According to some theories ice segregation should stop at certain pressures. The data from this boring suggests that even 146 ft of cover did not prevent ice segregation, assuming that the entire overburden was present at the time of freezing and that the ice-filled crevasses were not formed by some other means. The existence of this ice also emphasizes a common mistake which engineers unfamiliar with permafrost make—the assumption that permafrost problems are automatically avoided if foundations can be placed on bedrock. Numerous cases have been reported from Alaska and from Greenland in which substantial amounts of ice have been found in bedrock and which could produce very detrimental settlement if a heat-producing structure were founded on them and the ice were caused to thaw.

DYNAMIC TESTING OF PAVEMENT[a]

Discussion by A. A. Eremin

A. A. EREMIN,[33] M. ASCE.—Messrs. Heukelom and Foster correctly stated that the dynamic method of testing of pavement slabs and subgrade material is not destructive. Furthermore, the equipment for dynamic testing is more mobile than that used in the static tests. Therefore, the dynamic testing may conveniently be used in finding the errors in the roadway pavement and subgrade construction. Information available at present is still not adequate for design of pavement and subgrade based on the dynamic tests.

Velocity of the dynamic wave propagation in the pavement varies with the elastic properties of pavement and subgrade. Furthermore, the dynamic force exerted by the vibrator is in proportion to the frequencies of vibration. Therefore, the resulting velocity graphs of wave propagations are varied according to the stepped curve similar to those shown in Figs. 13 and 14. Obviously, variations in the thickness of pavement and depth of subgrade may be detected by observing the velocity graphs. The information from the velocity graphs, however, is not conclusive for design purposes.

It would be interesting if the authors would give more illustrative examples from application of dynamic testing of pavement. Likewise it would be interesting if the method of numerical evaluation of the damping force in Eq. 15 were shown.

In view of the novelty of the dynamic method of testing of pavement, the authors deserve high credit for presenting this most interesting paper.

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[a] February, 1960, by W. Heukelom and C. R. Foster.

[33] Assoc. Bridge Engr., Calif. State Highways, Sacramento, Calif.

SOIL STRUCTURE AND THE STEP-STRAIN PHENOMENA[a]

Discussion by D. F. Coates

D. F. COATES[18]—The authors are to be congratulated on the empirical work that has been done to substantiate their hypothesis of clay structure.

Recently, (1959) some work was done in Ottawa that involved using tests similar to those described in the paper on the local clay. The deposit consisted of a stiff, highly sensitive (up to 200 from field vane tests), highly compressible (average compression index 1.4), preconsolidated clay (average preconsolidation load 4 tsf). An upper layer was highly plastic with an average liquid limit of 55% and an average liquidity index of 1.2. A lower layer was of low placticity with an average liquid limit of 30 and an average liquidity index of 2.3. The results of the tests on these clays do not seem to substantiate the hypothesis on structure presented by the authors. For example, with failure strains of the order of 1% to 2% the stress-strain curves were of types I and II, as illustrated by the authors. This behavior is not in accordance with the classification of the curves given by the authors. Needless to say, it is not quite in accordance with the behavior which most soil engineers would expect from clay with such high sensitivities.

In addition, the incremental stress tests, in some cases, showed an erratic behavior such as illustrated in Fig. 13(a), however, in other cases the plot of incremental strain with level of stress produced a fairly straight line up to a definitive knee, such as Housel has been obtaining for many years with the Detroit clays. On the other hand, the plots of instantaneous cumulative strain versus stress all produced remarkably straight lines. Furthermore, static creep tests showed no step behavior such as described by the authors.

There was no obvious difference in the behavior of the two different layers of clay to these tests. The pore pressures in all tests were almost exactly proportional to the deviator stress. From these tests, as well as a controlled strain rate series, it was concluded that the maximum reduction in strength (in terms of total stresses) resulting from the passage of time would be between 10% and 25%. Consequently, it can be seen that the evidence from our tests would not seem to fulfill the requirements of either a step-strain clay or a pure matrix clay.

It is appreciated that it is difficult for the authors to comment on these statements without being able to examine all the details of the tests. The writer is preparing a paper that will include this information. Meanwhile, any explanations that the authors could offer on the following questions and comments would be most useful.

The authors suggest that the term "cohesion" be replaced by the term "colloidal friction." The objective of this change in definition is to be able to describe shear strength as being entirely derived from friction. The writer's

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[a] April, 1960, by D. H. Trollope and C. K. Chan.

[18] D. F. Coates Ltd., Cons. Engrs., Ottawa, Ontario, Canada.

reaction to this suggestion is that it is inaccurate because the term friction implied, on a microscopic scale, that the frictional resistance is a function of the normal force. With a constant void ratio, it would not be expected that this "colloidal friction" would vary with the magnitude of the normal stress.

Is there any substantiation for the assumption that the coarser grains flow towards the zone of plastic yield? It would seem that other more plausible mechanisms, such as stress concentrations plus progressive failures, could be imagined—both of which being conjectural are not as important as the actual behavior laws as empirically determined.

In order to explain the loss of strength due to time, the authors suggest that large, local volume changes with consequent high, local pore pressures might be significant. This does not strike the writer as being reasonable, for it is imagined that the immediate development of a deficiency of pore pressure in the zone of high shear strain with the ultimate dissipation of these possibly negative pore pressures with time would be a more probable sequence. As was mentioned previously, the test data that was obtained here did not substantiate either postulate. Have the authors any test data on the actual pore pressures to apply to this question?

As a criteria the authors that "after a condition of matrix yield has been reached in any specimen, the incremental deformation curve follows a more or less erratic pattern." This seems a vague and rather arbitrary criterion, and its application in Figs. 13 and 21 seems to support this comment.

Fig. 22 indicates that there are no steps in the step-strain clay below 60% of the normal unconfined compression strength. In the incremental stress-tests, steps were obtained at 10% to 30% of the normal unconfined compression strength. Could the authors suggest why steps should not have been obtained at lower levels of stress in the creep tests?

In conclusion, the writer would like to emphasize again the value of this paper. Such findings as 1) the greater strain resulting from repeated loads than sustained loads in some circumstances, and 2) the greater sensitivity of step-strain clays to a time reduction in strength-are extremely valuable, and it will probably be some time before their full implications are understood. As a final question, have the authors considered including in the structure of clays the presence of bound water films together with hydrogen bonding between the water films as a contributer to cohesion?

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the symbols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper number are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 2270 is identified as 2270(ST9) which indicates that the paper is contained in the ninth issue of the Journal of the Structural Division during 1959.

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AUGUST: 2126(HY8), 2127(HY8), 2128(HY8), 2129(HY8), 2130(PO4), 2131(PO4), 2132(PO4), 2133(PO4), 2134(SM4), 2135(SM4), 2136(SM4), 2137(SM4), 2138(HY8)[c], 2139(PO4)[c] 2140(SM4)[c].

SEPTEMBER: 2141(CO2), 2142(CO2), 2143(CO2), 2144(HW3), 2145(HW3), 2146(HW3), 2147(HY9), 2148(HY9), 2149(HY9), 2150(HY9), 2151(IR3), 2152(ST7)[c], 2153(IR3), 2154(IR3), 2155(IR3), 2156(IR3), 2157(IR3), 2158(IR3), 2159(IR3), 2160(IR3), 2161(SA5), 2162(SA5), 2163(ST7), 2164(ST7), 2165(SU1), 2166(SU1), 2167(WW3), 2168(WW3), 2169(WW3), 2170(WW3), 2171(WW3), 2172(WW3), 2173(WW3), 2174(WW3), 2175(WW3), 2176(WW3), 2177(WW3), 2178(CO2)[c], 2179(IR3)[c], 2180(HW3)[c], 2181(SA5)[c], 2182(HY9)[c], 2183(SU1)[c], 2184(WW3)[c], 2185(PP2)[c], 2186(ST7)[c], 2187(PP2), 2188(PP2).

OCTOBER: 2189(AT4), 2190(AT4), 2191(AT4), 2192(AT4), 2193(AT4), 2194(EM4), 2195(EM4), 2196(EM4), 2197(EM4), 2198(EM4), 2199(EM4), 2200(HY10), 2201(HY10), 2202(HY10), 2203(PL3), 2204(PL3), 2205(PL3), 2206(PO5), 2207(PO5), 2208(PO5), 2209(PO5), 2210(SM5), 2211(SM5), 2212(SM5), 2213(SM5), 2214(SM5), 2215(SM5), 2216(SM5), 2217(SM5), 2218(ST8), 2219(ST8), 2220(EM4), 2221(ST8), 2222(ST8), 2223(ST8), 2224(HY10), 2225(HY10), 2226(PO5), 2227(PO5), 2228(PO5), 2229(ST8), 2230(EM4), 2231(EM4), 2232(AT4)[c], 2233(PL3)[c], 2234(EM4)[c], 2235(HY10)[c], 2236(SM5)[c], 2237(ST8)[c], 2238(PO5)[c], 2239(ST8), 2240(PL3).

NOVEMBER: 2241(HY11), 2242(HY11), 2243(HY11), 2244(HY11), 2245(HY11), 2246(SA6), 2247(SA6), 2248(SA6), 2249(SA6), 2250(SA6), 2251(SA6), 2252(SA6), 2253(SA6), 2254(SA6), 2255(SA6), 2256(ST9), 2257(ST9), 2258(ST9), 2259(ST9), 2260(HY11), 2261(ST9)[c], 2262(ST9), 2263(HY11), 2264(ST9), 2265(HY11), 2266(SA6), 2267(SA6), 2268(SA6), 2269(HY11)[c], 2270(ST9).

DECEMBER: 2271(HY12)[c], 2272(CP2), 2273(HW4), 2274(HW4), 2275(HW4), 2276(HW4), 2277(HW4), 2278(HW4), 2279(HW4), 2280(HW4), 2281(IR4), 2282(IR4), 2283(IR4), 2284(IR4), 2285(PO6), 2286(PO6), 2287(PO6), 2288(PO6), 2289(PO6), 2290(PO6), 2291(PO6), 2292(SM6), 2293(SM6), 2294(SM6), 2295(SM6), 2296(SM6), 2297(WW4), 2298(WW4), 2299(WW4), 2300(WW4), 2301(WW4), 2302(WW4), 2303(WW4), 2304(HW4), 2305(ST10), 2306(CP2), 2307(CP2), 2308(ST10), 2309(CP2), 2310(HY12), 2311(HY12), 2312(PO6), 2313(PO6), 2314(ST10), 2315(HY12), 2316(HY12), 2317(HY12), 2318(WW4), 2319(SM6), 2320(SM6), 2321(ST10), 2322(ST10), 2323(HW4)[c], 2324(CP2)[c], 2325(SM6)[c], 2326(WW4)[c], 2327(IR4)[c], 2328(PO6)[c], 2329(ST10)[c], 2330(CP2).

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JANUARY: 2331(EM1), 2332(EM1), 2333(EM1), 2334(EM1), 2335(HY1), 2336(HY1), 2337(EM1), 2338(EM1), 2339(HY1), 2340(HY1), 2341(SA1), 2342(EM1), 2343(SA1), 2344(ST1), 2345(ST1), 2346(ST1), 2347(ST1), 2348(EM1)[c], 2349(HY1)[c], 2350(ST1), 2351(ST1), 2352(SA1)[c], 2353(ST1)[c], 2354(ST1).

FEBRUARY: 2355(CO1), 2356(CO1), 2357(CO1), 2358(CO1), 2359(CO1), 2360(CO1), 2361(PO1), 2362(HY2), 2363(ST2), 2364(HY2), 2365(SU1), 2366(HY2), 2367(SU1), 2368(SM1), 2369(HY2), 2370(SU1), 2371(HY2), 2372(PO1), 2373(SM1), 2374(HY2), 2375(PO1), 2376(HY2), 2377(CO1)[c], 2378(SU1), 2379(SU1), 2380(SU1), 2381(HY2)[c], 2382(ST2), 2383(SU1), 2384(ST2), 2385(SU1)[c], 2386(SU1), 2387(SU1), 2388(SU1), 2389(SM1), 2390(ST2)[c], 2391(SM1)[c], 2392(PO1)[c].

MARCH: 2393(IR1), 2394(IR1), 2395(IR1), 2396(IR1), 2397(IR1), 2398(IR1), 2399(IR1), 2400(IR1), 2401(IR1), 2402(IR1), 2403(IR1), 2404(IR1), 2405(IR1), 2406(IR1), 2407(SA2), 2408(SA2), 2409(HY3), 2410(ST3), 2411(SA2), 2412(HW1), 2413(WW1), 2414(WW1), 2415(HY3), 2416(HW1), 2417(HW3), 2418(HW1)[c], 2419(WW1)[c], 2420(WW1), 2421(WW1), 2422(WW1), 2423(WW1), 2424(SA2), 2425(SA2)[c], 2426(HY3)[c], 2427(ST3)[c].

APRIL: 2428(ST4), 2429(HY4), 2430(PO2), 2431(SM2), 2432(PO2), 2433(ST4), 2434(EM2), 2435(PO2), 2436(ST4), 2437(ST4), 2438(HY4), 2439(EM2), 2440(EM2), 2441(ST4), 2442(SM2), 2443(HY4), 2444(ST4), 2445(EM2), 2446(ST4), 2447(EM2), 2448(SM2), 2449(HY4), 2450(ST4), 2451(HY4), 2452(HY4), 2453(EM2), 2454(EM2), 2455(EM2)[c], 2456(HY4)[c], 2457(PO2)[c], 2458(ST4)[c], 2459(SM2)[c].

MAY: 2460(AT1), 2461(ST5), 2462(AT1), 2463(AT1), 2464(CP1), 2465(CP1), 2466(AT1), 2467(AT1), 2468(SA3), 2469(HY5), 2470(ST5), 2471(SA3), 2472(SA3), 2473(ST5), 2474(SA3), 2475(ST5), 2476(SA3), 2477(ST5), 2478(HY5), 2479(SA3), 2480(ST5), 2481(SA3), 2482(CO2), 2483(CO2), 2484(HY5), 2485(HY5), 2486(AT1)[c], 2487(CP1)[c], 2488(CO2)[c], 2489(HY5)[c], 2490(SA3)[c], 2491(ST5)[c], 2492(CP1), 2493(CO2).

JUNE: 2494(IR2), 2495(IR2), 2496(ST6), 2497(EM3), 2498(EM3), 2499(EM3), 2500(EM3), 2501(SM3), 2502(EM3), 2503(PO3), 2504(WW2), 2505(EM3), 2506(HY6), 2507(WW2), 2508(PO3), 2509(ST6), 2510(EM3), 2511(EM3), 2512(ST6), 2513(HW2), 2514(HY6), 2515(PO3), 2516(EM3), 2517(WW2), 2518(WW2), 2519(EM3), 2520(PO3), 2521(HY6), 2522(SM3), 2523(ST6), 2524(HY6), 2525(HY6), 2526(HY6), 2527(IR2), 2528(ST6), 2529(HW2), 2530(IR2), 2531(HY6), 2532(EM3)[c], 2533(HW2)[c], 2534(WW2), 2535(HY6)[c], 2536(IR2)[c], 2537(PO3)[c], 2538(SM3)[c], 2539(ST6)[c], 2540(WW2)[c].

JULY: 2541(ST7), 2542(ST7), 2543(SA4), 2544(ST7), 2545(ST7), 2546(HY7), 2547(ST7), 2548(SU2), 2549(SA4), 2550(SU2), 2551(HY7), 2552(ST7), 2553(SU2), 2554(SA4), 2555(ST7), 2556(SA4), 2557(SA4), 2558(SA4), 2559(ST7), 2560(SU2)[c], 2561(SA4)[c], 2562(HY7)[c], 2563(ST7)[c].

AUGUST: 2564(SM4), 2565(EM4), 2566(ST8), 2567(EM4), 2568(PO4), 2569(PO4), 2570(HY8), 2571(EM4), 2572(EM4), 2573(EM4), 2574(SM4), 2575(EM4), 2576(EM4), 2577(HY8), 2578(EM4), 2579(PO4), 2580(EM4), 2581(ST8), 2582(ST8), 2583(EM4)[c], 2584(PO4)[c], 2585(ST8)[c], 2586(SM4)[c], 2587(HY8)[c].

c. Discussion of several papers, grouped by divisions.

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PART 2

AUGUST 1960 — 28 NO. SM 4
VOLUME 86 PART 2

Your attention is invited

NEWS OF THE SOIL MECHANICS AND FOUNDATIONS DIVISION OF ASCE

JOURNAL OF THE SOIL MECHANICS AND FOUNDATIONS DIVISION
PROCEEDINGS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS

DIVISION ACTIVITIES

SOIL MECHANICS AND FOUNDATIONS DIVISION

Proceedings of the American Society of Civil Engineers

NEWS

August, 1960

NEWS FROM THE WEST COAST

Meetings and Special Events Held in the San Francisco Bay Area Included the Following:

On May 3, Port of Oakland sponsored, for the Air Transport Division of the San Francisco Section, a tour of the $17,5000,000 construction project at the Oakland Metropolitan International Airport. The project involves reclamation of 600 acres of tidal mud-flat from San Francisco Bay for the development of a new jet runway, 12,000 feet long, a passenger terminal building, and related taxiways, ramps, aprons, and service areas. Some 14,500,000 cubic yards of sand were dredged in from selected pits in San Francisco Bay. The sand was placed by a 30" hydraulic dredge belonging to Utah Construction Company.

On May 12, at a meeting of the Soil Mechanics and Foundations Division, Dr. G. C. Meyerhof, Professor of Civil Engineering, Novia Scotia, Technical College, Canada, spoke on "Some Research on the Bearing Capacity of Foundations," a subject in which he is eminently qualified.

On May 25, Dr. A. W. Bishop, Imperial College, London University, addressed the local Soil Mechanics and Foundations Division on the subject "The Role of Field Measurements of Pore Pressure in Earth Dam Construction." Dr. Bishop's talk concerned itself, for the most part with construction of Selset Dam in England; the design of which involved use of vertical sand drains for reducing construction pore pressures in the alluvial foundation strata and horizontal layers in the embankment for relief of pore pressures developed by constructing the embankment at water contents higher than the optimum.

NEWS FROM TEXAS

The new officers of the Midland Texas Section are as follows: Chairman, Dr. Lymon C. Reese, Mr. Raymond C. Mason, Mason-Johnston and Associates, Vice President, and Professor Cecil H. Smith, Southern Methodist University, Secretary.

Four speakers presented programs at the first meeting of the new regime.

Note.—No. 1960-28 is part of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. SM 4, August, 1960.

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The first speaker was Mr. Francis M. Bell, U.S.G.S. of Denver, Colorado, who spoke and showed slides of the effects of the recent Yellowstone earthquake. Of particular interest was the strength and flexibility of the earthern Hebgen Dam which successfully withstood the shear and impact forces of foundation settlements and wave action over the dam. The next speaker was Mr. Arthur Guy of the Humble Oil and Refining Company, who spoke on the soil studies and design procedures for weights, backfill, and earth anchors used to combat buoyancy of a large gas pipline laid in marshy land. The third speaker, Mr. Clyde R. Butler, of the Atlantic Refining Company, who spoke of laboratory and field experiences with excessive leaking of brine pits from the action of concentrated salts, and the use of polyethylene linings. The last speaker was Mr. Bobby L. Myers of the University of Texas, who gave the results of an investigation of a rapid consolidation test by curve fitting to an ideal time-consolidation curve.

SYMPOSIUM ON LOAD BEARING CAPACITY OF SOILS

The Central Building Research Institute, Roorkee (U.P.) India proposes to hold a symposium, probably at Roorkee, but possibly at New Delhi towards the end of January 1961.

The symposium will deal with the following subjects:

1. Field bearing capacity tests and their interpretation.
2. Laboratory bearing capacity tests and their interpretation.
3. Settlements, their estimation, and permissible allowances.
4. Various theories on Bearing Capacities and their application to design.

It is planned to invite some eminent soils engineers from abroad, who will present papers and participate in the symposium. For further information, contact: Mr. H. Williams, Director.

FIFTH INTERNATIONAL SOILS CONFERENCE
Charter Flights

The U. S. National Committee for the International Society on Soil Mechanics and Foundations Engineering together with the U. S. Committee on Large Dams are investigating the possibilities of charter flights to the Fifth International Conference on Soil Mechanics and Foundations Engineering to be held in Paris in 1961 and the Seventh Congress on Large Dams to be held in Rome in 1961. Dates of tours and meetings of these conferences are as follows:

7th Congress on Large Dams

June 26 through July 1	Meetings	Rome
July 2 through July 9	Tours	Italy

5th International Conference on Soil Mechanics and Foundations Engineering

July 10 through July 14	Tours	France
July 17 through July 22	Meetings	Paris
July 24 through July 30	Tours	France

The charter flights would probably be in DC-7 propeller aircraft. For economy configuration the cost per person would be roughly one half of economy class on jet flights. Meals would be similar to meals served on first

class. Wives and children could be carried on the charter flights as well as members of the sponsoring organization. Also, if the member must travel by other means of transportation and is not able to use the charter flight, his wife and children may still do so. A deposit of 10-20% for reservations on the charter flights would have to be made about November 1960 and final payment about two months in advance of the flight.

To permit planning, would all persons interested in such charter flights please return the questionnaire given on the last page before September 15th to John Lowe, III, Secretary, U. S. National Committee for Intl. Soc. Soil Mech. & Found. Engrg., 375 Park Avenue, Room 900, New York 22, N. Y. In case of oversubscription to flights, persons returning this questionnaire will be given preference.

COMBINED INDEX TO ASCE PUBLICATIONS

For complete coverage of the Society's 1959 year in print, there is now a Combined Index covering the Division Journals, Transactions, and Civil Engineering. Also included are reprints of the Proceedings Abstracts that are published each month in Civil Engineering. The price of the Combined Index (ASCE publication 1960-10) is $2.00 with the usual 50% discount to members. The coupon herewith will make ordering easy:

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. . . of 1960-10

Alternative Flights	Dates Going and Returning	Round Trip Cost/Person Economy (Circle One)*	First Class	Indicate No. of Persons	Order of Preference (Circle One)*	Probability of Using (Circle One)*
#1						
17 days	Lv New York June 24	$320	$470	___	1	Quite Sure
	Ar Rome June 25				2	50 - 50 Probability
					3	Slight Probability
	Lv Rome July 9				4	
	Ar New York July 10					
#2						
23 days	Lv New York June 24	$290	$420	___	1	Quite Sure
	Ar Rome June 25				2	50 - 50 Probability
					3	Slight Probability
	Lv Paris July 15				4	
	Ar New York July 16					
#3						
25 days	Lv New York July 8	$260	$380	___	1	Quite Sure
	Ar Paris July 9				2	50 - 50 Probability
					3	Slight Probability
	Lv Paris July 31				4	
	Ar New York Aug. 1					
#4						
39 days	Lv New York June 24	$290	$420	___	1	Quite Sure
	Ar Rome June 25				2	50 - 50 Probability
					3	Slight Probability
	Lv Paris July 31				4	
	Ar New York Aug. 1					

*Circle one for each alternative flight

OCTOBER NEWSLETTER

Deadline date for arrival at this office of contributions for the October Newsletter: August 26, please.

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INTERNATIONAL CLASSIFICATION SYSTEM
MODIFIED PROPOSAL Number 2

The following material has been received from Mr. Nils Flodin, Secretary of the Sub-Committee for the classification of Geotechnical Literature of the International Congress of Soil Mechanics and Foundation Engineering. This sub-committee has been engaged in this work for a number of months. It is composed of the following men:

Mr. Nils Flodin
Professor ir E. de Beer
Dr. L. Bjerrum
Mr. Osterman
Professor Petermann
Professor A. Casagrande
Professor A. Croce
Dr. Cooling
Professor Skempton
Professor Kerisel
Mr. Banister

Principal Sections

0 GENERAL

1 ENGINEERING GEOLOGY

2 SITE EXPLORATION, SAMPLING, AND FIELD MEASUREMENTS

3 ENGINEERING PROPERTIES OF SOILS AND LABORATORY TESTING

4 WATER AND CLIMATIC PROBLEMS

5 SETTLEMENTS AND OTHER STRESS-DEFORMATION PROBLEMS

6 FAILURE PROBLEMS AND EARTH PRESSURE

7 CONSTRUCTION METHODS, FOUNDATION AND EARTH WORKS

8 DESCRIPTION OF PROJECTS

9 MISCELLANEOUS PROBLEMS AND WORKS

0 GENERAL

00 Definition and Scope

01 History and Biographies

02 Nomenclature and Symbols

03 Societies, Conferences, Education

04 Institutions, Firms, etc.

05 Bibliographies, Incl. Classification of Literature

06 General Textbooks and Handbooks

07 Codes of Practice, etc.

08 Auxiliary Literature of Various Sciences

09 Miscellaneous

1 ENGINEERING GEOLOGY

10 General

11 Mineralogy and Physico-Chemical Fundamentals of Soil Constituents, incl. Laboratory Investigations

12 Soil Origin, Transportation and Deposition

13 General Description of Soils, incl. Genetic and Similar Classification

14 Changes of Soil after Deposition and Natural Mass Movements

15 Regional Soil Conditions and Problems

16 Engineering Rock Geology
Rock classification, faults, vulcaneology, etc.
Rock mechanics and rock foundation engineering, see 91

17 Regional Rock Conditions and Problems

18

19 Miscellaneous
For instance, land heave and sinking
Water fluctuations

2 SITE EXPLORATION, SAMPLING, AND FIELD MEASUREMENTS

20 General

21 Inspection of Site, incl. Biological and Airphoto Studies, etc.

22 Geophysical Explorations

23 Mechanical Probing (Soundings, etc.)

24 Test Pits and Test Tunnels

25 Making of Bore Holes (Boring Technique)

26 Sampling and Handling of Samples

27 Field Measurements of Deformations and Stresses (incl. Methods of Loading Tests and Pile Testing, etc.)

28 Other Field Measurements, incl. Hydrologic and Pore Pressure Measurements

29 Miscellaneous

3 ENGINEERING PROPERTIES OF SOILS AND LABORATORY TESTING

30 General

31 Identification, Classification and Index Properties Physico-Chemical fundamentals of soil constituents, see 11

32 Viscosity, Thixotropy and Other Rheologic Fundamentals Deformation properties, see 35; strength properties, see 36

33 Permeability and Capillarity

34 Freezing and Thermal Properties

35 Consolidation and Other Deformation Properties

36 Shearing Characteristics and Strength Properties

37 Compaction Properties, incl. Vibration Effects, and Special Compaction Tests
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38 Stabilization Properties, incl. Mechanical Properties of Stabilized Soils

39 Miscellaneous

4 WATER AND CLIMATIC PROBLEMS

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42 Natural Seepage Problems (Excl. Erosion)

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48

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64 Bearing Capacity of Spread Foundations and Other Surface Load Problems

65 Bearing Capacity of Piles

66 Failure Under Dynamic Load (incl. Earth Quakes)

67 Safety Problems, incl. Special Calculations of Probability

68

69 Miscellaneous

7 CONSTRUCTION METHODS

Foundation and Earth Works

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(Incl. Costs of Construction and Maintenance)

71 Spread Foundation

72 Pile Foundation

73 Pier and Caisson Foundation

74 Retaining Structures, incl. Sheeting and Bracing, Bulkheads and Cofferdams

75 Tunnelling and Conduit Works

76 Earth and Rockfill Embankments Construction, incl. Excavation and Transport Processes.
Road and Airfield Construction
Compaction works, see 77

77 Improvement of Soils, incl. Compaction and Stabilization methods

78 Protection Works against Water. Drainage Works

79 Miscellaneous
(Incl. General Construction Machinery)

8 DESCRIPTION OF PROJECTS

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82 Bridge Piers and Retaining Structures

83 Waterfront Structures (Harbours Projects, etc.)

84 Waterways

85 Tunnels and Conduits

86 Dams, Embankments, and Other Fillings

87 Roads, Airfields, and Railways

88

89 Miscellaneous
(Incl. Projects in Connection with Region and Town Planning)

9 MISCELLANEOUS PROBLEMS AND WORKS

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92 Snow and Ice Mechanics

93 Properties of Other Filling Masses than Soils

94 Deterioration, Corrosion and Preservation of Construction Materials

95 Damage of Structures due to Geotechnical Reasons